8. Augustine achieves heaven if Augustine is virtuous. But Augustine is happy provided that he is not virtuous. Augustine does not achieve heaven only if he is not happy. Therefore, Augustine achieves heaven. (A: Augustine achieves heaven; V: Augustine is virtuous; H: Augustine is happy)

9. Not all living things are able to feel pain. For all living things are able to feel pain only if all living things have nervous systems. But not all living things have nervous systems given that plants do not have nervous systems. And plants do not have nervous systems. (L: All living things are able to feel pain; N: All living things have nervous systems; P: Plants have nervous systems)

10. It is morally permissible for mentally superior extraterrestrials to eat humans on the condition that it is morally permissible for humans to eat animals. But either it is not morally permissible for mentally superior extraterrestrials to eat humans, or human life lacks intrinsic value. However, human life has intrinsic value. We are forced to conclude that it is not morally permissible for humans to eat animals. (E: It is morally permissible for mentally superior extraterrestrials to eat humans; H: It is morally permissible for humans to eat animals; V: Human life has intrinsic value)

7.4 Abbreviated Truth Tables

As we have seen, the truth table method is rather cumbersome when applied to arguments having more than three statement letters. But there are ways to make it less cumbersome, and we will explore one of them in this section, namely, the **abbreviated truth table method**. The essential insight behind abbreviated truth tables is this: If we can construct *one* row of a truth table, *making all the premises true while the conclusion is false*, then we have shown that the argument form in question is invalid. Here's an example:

85. If I am thinking, then my neurons are firing. Hence, if my neurons are firing, then I am thinking. (A: I am thinking; N: My neurons are firing)

Using the scheme of abbreviation provided, we may symbolize the argument as follows:

86. $A \rightarrow N \therefore N \rightarrow A$

We begin by hypothesizing that all the argument's premises are true while its conclusion is false:

	$A \rightarrow N$	$\therefore$	$N \rightarrow A$
	T		F

Now we work backward. If the conclusion is false, then *N* must be true and *A* must be false. We fill in these values uniformly throughout the argument:

	A → N	∴	N → A
	F T T		T F F

This truth assignment does indeed make the conclusion false and the premise true. We have in effect constructed a row in the truth table that shows the argument to be invalid: It is the row in which *A* is false and *N* is true. We add this information at the left to complete our abbreviated truth table:

A N	A → N	∴	N → A
F T	F T T		T F F

We have thus shown the argument to be invalid. And as before, our truth-functional assessment of the argument gives a strong hint about how to construct an English counterexample:

87. If Thomas Jefferson was 500 years old when he died, then he lived to be more than a year old. Therefore, if Jefferson lived to be more than a year old, then he was 500 years old when he died.

The premise of argument (87) is plainly true, even though its antecedent is false. And, of course, the conclusion of the argument is false, too.

Let's try a more complicated example. Consider the following symbolic argument:

88. E v S, E → (B • U), ~S v ~U ∴ B

Again, we hypothesize that all the premises can be true while the conclusion is false:

	E v S,	E → (B • U),	~S v ~U	∴	B
	T	T	T		F

Then we work backward to determine the truth value of each constituent statement letter. Since we have assigned "F" to *B* in the conclusion, we must assign "F" to *B* uniformly throughout the argument. (Remember, we are in effect constructing a single row in a truth table.) This means that *B* • *U* is also false. Hence, we must assign "F" to *E*; otherwise, the second premise will be false, which contradicts our hypothesis. Now, if *E* is false, we must make *S* true;

otherwise, the first premise will be false, which contradicts our hypothesis. And if *S* is true, then $\sim S$ is false, so we must make $\sim U$ true (and hence *U* false) to make the third premise true. Thus, we arrive at our abbreviated truth table:

E	S	B	U	$E \vee S$,	$E \rightarrow (B \bullet U)$,	$\sim S \vee \sim U$	$\therefore$ B
F	T	F	F	F T T	F T F F F	F T T T F	F

In this case, an argument that would require a 16-row truth table can be dealt with quickly by means of an abbreviated truth table.

The abbreviated truth table method can also be used to show that an argument is valid. Let's try it out on an old friend, disjunctive syllogism. Again, we begin by hypothesizing that the conclusion can be false while the premises are true:

	$A \vee B$,	$\sim A$	$\therefore$ B
	T	T	F

If $\sim A$ is true, then *A* is false. But since *B* is also false, $A \vee B$ is false, contrary to our hypothesis. Thus, in trying to assign values so that the premises are all true and the conclusion is false, we are forced to contradict ourselves. This means the argument is valid. We indicate that we were forced to assign truth values inconsistently by writing "T/F" under the first premise:

	$A \vee B$,	$\sim A$	$\therefore$ B	
	F T/F F	T F	F	Valid

Using an abbreviated truth table is a bit more complicated when the conclusion of the argument is false on *more than one* assignment of truth values—for example, when the conclusion is a conjunction or a biconditional. In such cases, the following principles will suffice:

> **Principle 1:** If there is *any* assignment of values in which the premises are all true and the conclusion is false, then the argument is invalid.
>
> **Principle 2:** If more than one assignment of truth values will make the conclusion false, then consider each such assignment; if each assignment that makes the conclusion false makes *at least one* premise false, then the argument is valid.

For instance, consider the following symbolic argument:

89. $F \rightarrow G$, $G \rightarrow H$ $\therefore$ $\sim F \bullet H$

There are three ways to make the conclusion false: (a) make both conjuncts false, (b) make the left conjunct false and the right one true, or (c) make the left conjunct true and the right one false. If we neglect this complexity, we can easily fall into error, for not every assignment that makes the conclusion false makes the premises true. For instance:

	$F \rightarrow G$,	$G \rightarrow H$	$\therefore$	$\sim F \bullet H$
	T T/F F	F T F		F T F F

With this assignment, the first premise is false. (We could make the first premise true by assigning "T" to G, but then the second premise would be false.) If we overlook the fact that other truth value assignments render the *conclusion* false, we might suppose that this abbreviated truth table shows that the argument is valid. But it does not, because there is a way of assigning "F" to the conclusion that makes all the premises true, namely:

F	G	H	$F \rightarrow G$,	$G \rightarrow H$	$\therefore$	$\sim F \bullet H$
F	F	F	F T F	F T F		T F F F

And this proves that the argument form is invalid.

To show that an argument is valid, we must consider every truth value assignment in which its conclusion is false. Consider the following example:

90. $P \rightarrow Q,\ Q \rightarrow P\ \therefore\ P \leftrightarrow Q$

A biconditional is false whenever its two constituent statements *differ* in truth value. So, in this case, we must consider the assignment in which *P* is true and *Q* is false, *and also* the assignment in which *P* is false and *Q* is true.

	$P \rightarrow Q$,	$Q \rightarrow P$	$\therefore$	$P \leftrightarrow Q$	
	T T/F F	F T T		T F F	
	F T T	T T/F F		F F T	Valid

Here, each assignment that makes the conclusion false also makes one of the premises false (which contradicts our hypothesis that all the premises can be true while the conclusion is false). Thus, we have shown the argument to be valid.

The following exercise gives you an opportunity to practice the abbreviated truth table method.

Exercise 7.4

Part A: Abbreviated Truth Tables Use abbreviated truth tables to show that the following arguments are invalid.

* 1. $A \rightarrow (B \rightarrow C) \therefore B \rightarrow C$

2. $\sim(E \leftrightarrow F) \therefore \sim E \bullet \sim F$

3. $\sim(G \leftrightarrow H) \therefore \sim G \rightarrow \sim H$

* 4. $J \rightarrow \sim K \therefore \sim(J \leftrightarrow K)$

5. $(P \bullet Q) \rightarrow R,\ \sim R \therefore \sim P$

6. $\sim(Z \bullet H), \sim Z \rightarrow Y, W \rightarrow H \therefore \sim W \rightarrow Y$
   How many rows would be needed in a complete truth table for argument 6?

* 7. $\sim(S \bullet H), (\sim S \bullet \sim H) \rightarrow \sim U \therefore \sim U$

8. $(F \bullet G) \leftrightarrow H, \sim H \therefore \sim G$

9. $\sim(B \rightarrow C), (D \bullet C) \vee E \therefore \sim B$

* 10. $(P \rightarrow \sim Q) \leftrightarrow \sim R, R \therefore \sim P$

11. $S \rightarrow (T \rightarrow V) \therefore (S \rightarrow T) \rightarrow V$

12. $A \rightarrow (B \rightarrow C) \therefore A \rightarrow (B \bullet C)$

* 13. $(Z \bullet Y) \rightarrow W \therefore Z \rightarrow (Y \bullet W)$

14. $\sim(C \vee D), (\sim C \bullet \sim E) \leftrightarrow \sim D, \sim E \rightarrow (C \vee F), S \vee F \therefore S$
   How many rows would be needed in a complete truth table for argument 14?

15. $(F \leftrightarrow G) \leftrightarrow H, \sim H \therefore \sim F \bullet \sim G$

* 16. $P \rightarrow Q, P \rightarrow R, Q \leftrightarrow R, S, S \rightarrow R \therefore P \bullet Q$

17. $S \rightarrow (A \bullet O), \sim P \vee \sim R, P \rightarrow (S \vee Z), Z \rightarrow (O \rightarrow R) \therefore Z \vee \sim P$

18. $A \vee (B \bullet C), \sim A \therefore (A \bullet B) \vee (A \bullet C)$

* 19. $\sim(Q \vee S), \sim T \vee S, (U \bullet W) \rightarrow Q \therefore (\sim T \bullet \sim U) \bullet W$

20. $\sim J \bullet \sim K, L \rightarrow J, M \rightarrow K, (M \rightarrow \sim L) \rightarrow \sim(N \bullet O) \therefore \sim N$

Part B: More Abbreviated Truth Tables Use abbreviated truth tables to show that the following arguments are invalid.

* 1. $\sim(A \bullet B), \sim A \rightarrow C, \sim B \rightarrow D \therefore C \bullet D$

2. $L \leftrightarrow (M \bullet N), M \vee N, \sim L \therefore \sim M$

3. $(O \leftrightarrow P) \rightarrow R, \sim R \therefore \sim O \vee P$

* 4. $\sim(V \bullet X) \rightarrow \sim Y \therefore \sim[(V \bullet X) \rightarrow Y]$

5. $\sim(Z \bullet H), \sim Z \rightarrow Y, W \rightarrow H \therefore \sim W \rightarrow Y$

6. $\sim X \vee (C \bullet A), \sim Y \vee \sim B, \sim Y \vee (X \vee T), T \rightarrow (A \rightarrow B) \therefore T \vee \sim Y$

* 7. $\sim(Z \rightarrow A), Z \rightarrow B, \sim A \rightarrow C \therefore C \bullet \sim B$

8. $B \rightarrow (C \bullet D)$, $\sim E \vee \sim F$, $E \rightarrow (B \vee G)$, $G \rightarrow (D \rightarrow F)$ $\therefore$ $G \vee \sim E$
How many rows would be needed in a complete truth table for item 8?

9. $\sim(D \leftrightarrow E)$, $\sim D \rightarrow F$, $E \rightarrow G$ $\therefore$ $F \bullet G$

*10. $H \vee \sim S$, $H \rightarrow Z$, $\sim S \rightarrow P$ $\therefore$ $P \leftrightarrow Z$

11. $\sim[(J \bullet K) \rightarrow (M \vee N)]$ $\therefore$ $K \bullet N$

12. $A \rightarrow B$, $C \rightarrow \sim D$, $\sim B \vee D$ $\therefore$ $\sim A \leftrightarrow \sim C$

13. $\sim E \rightarrow (G \bullet A)$, $\sim(P \leftrightarrow \sim L) \vee E$, $\sim(P \bullet L) \vee Q$, $\sim N \rightarrow \sim G$ $\therefore$ $Q \bullet A$

14. $(G \rightarrow E) \leftrightarrow S$, $\sim(S \vee H)$, $\sim(P \bullet \sim H)$ $\therefore$ $G \bullet E$

15. $\sim(C \leftrightarrow \sim D) \vee E$, $\sim E \rightarrow (G \bullet H)$, $(C \bullet D) \rightarrow K$, $\sim N \rightarrow \sim G$ $\therefore$ $K \bullet H$
How many rows would be needed in a complete truth table for item 15?

Part C: Valid or Invalid? Some of the following arguments are valid, and some are invalid. Use abbreviated truth tables to determine which are valid and which are invalid.

* 1. $\sim A \vee B$ $\therefore$ $A \rightarrow B$

2. $F \rightarrow (G \leftrightarrow H)$, $\sim F \bullet \sim H$ $\therefore$ $\sim G$

3. $\sim M$ $\therefore$ $\sim N \vee \sim M$

* 4. $A \vee (B \bullet C)$ $\therefore$ $(A \bullet B) \vee (A \bullet C)$

5. $P \rightarrow \sim(Q \bullet R)$, $P \bullet R$ $\therefore$ $\sim Q$

6. $X \rightarrow Z$, $Y \rightarrow Z$, $\sim Z$ $\therefore$ $X \leftrightarrow Y$

7. $\sim(S \rightarrow R)$, $S \rightarrow J$, $\sim R \leftrightarrow W$ $\therefore$ $W \rightarrow \sim J$

8. $\sim M \rightarrow O$, $\sim N \rightarrow O$, $\sim O \leftrightarrow \sim P$, $\sim P$ $\therefore$ $M \bullet N$
How many rows would be needed in a complete truth table for item 8?

9. $(A \vee B) \bullet (A \vee C)$ $\therefore$ $A \bullet (B \vee C)$

10. $R \leftrightarrow \sim Q$, $R \vee Q$, $R \vee P$ $\therefore$ $(P \bullet Q) \rightarrow R$

Part D: English Arguments Translate the following English arguments into symbols, using the schemes of abbreviation provided. Use abbreviated truth tables to determine whether the arguments are valid.

* 1. If you want to mess up your life, you should drink a lot of booze. Therefore, if you don't want to mess up your life, you should not drink a lot of booze. (W: You want to mess up your life; B: You should drink a lot of booze)

2. Being undetermined is a necessary but not a sufficient condition for human behavior's being free. The laws of subatomic physics are statistical only if human behavior is not determined. And the laws of subatomic physics are statistical. It follows that human behavior is free. (D: Human behavior is determined; F: Human behavior is free; L: The laws of subatomic physics are statistical)

3. Given that nuclear energy is needed if and only if solar energy cannot be harnessed, nuclear energy is not needed. For solar energy can be harnessed provided that funds are available; and funds are available. (N: Nuclear energy is needed; S: Solar energy can be harnessed; F: Funds are available)

* 4. If the Gulf War was about oil, and if human life is more valuable than oil, then the Gulf War was immoral. Human life is more valuable than oil, but the Gulf War was not about oil. Therefore, the Gulf War was not immoral. (G: The Gulf War was about oil; H: Human life is more valuable than oil; I: The Gulf War was immoral)

5. The rate of teenage drunk driving will decrease just in case the taxes on beer increase. The taxes on beer increase only if either the federal government or the state government will resist the liquor lobby. The state government will resist the liquor lobby, but the federal government will not. Accordingly, the rate of teenage drunk driving will not decrease. (R: The rate of teenage drunk driving will decrease; B: The taxes on beer increase; F: The federal government will resist the liquor lobby; S: The state government will resist the liquor lobby)

6. Erik attains Valhalla given that he is valiant. And Erik is depressed assuming that he is not valiant. Furthermore, Erik fails to attain Valhalla only if he is not depressed. Thus, Erik is depressed. (E: Erik attains Valhalla; V: Eric is valiant; D: Eric is depressed)

* 7. If society is the ultimate source of moral authority, then if society approves of polygamy, polygamy is right. But it is not true either that society is the ultimate source of moral authority or that society approves of polygamy. Hence, polygamy is not right. (S: Society is the ultimate source of moral authority; P: Society approves of polygamy; R: Polygamy is right)

8. Either the earth is millions of years old, or it is only 6000 years old. If the earth is millions of years old, then the traditional story of creation is a myth, and ultimate reality is nothing but atoms in motion. Now, either it is false that the earth is only 6000 years old, or it is false that ultimate reality is nothing but atoms in motion. Therefore, the traditional story of creation is a myth. (E: The earth is millions of years old; S: The earth is only 6000 years old; B: The traditional story of creation is a myth; U: Ultimate reality is nothing but atoms in motion)

9. Wittgensteinians are right if logic is embedded in language. But logic is embedded in language if and only if logic varies as language varies. And logic is language-relative if logic varies as language varies. Moreover, given that logic is language-relative, contradictions may be true in some languages. Therefore, Wittgensteinians are right only if contradictions may be true in some languages. (W: Wittgensteinians are right; E: Logic is embedded in language; V: Logic varies as language varies; R: Logic is language-relative; C: Contradictions may be true in some languages)