EXERCISES

2 **3.3-1.** Let the random variable X have the p.d.f. $f(x) = 2(1 - x), 0 \le x \le 1$, zero elsewhere.

(a) Sketch the graph of this p.d.f.
(b) Determine and sketch the graph of the distribution function of X.
(c) Find (i) $P(0 \le X \le 1/2)$, (ii) $P(1/4 \le X \le 3/4)$, (iii) $P(X = 3/4)$, and (iv) $P(X \ge 3/4)$.

1 **3.3-2.** For each of the following functions, (i) find the constant c so that $f(x)$ is a p.d.f. of a random variable X, (ii) find the distribution function, $F(x) = P(X \le x)$, and (iii) sketch graphs of the p.d.f. $f(x)$ and the distribution function $F(x)$:

(a) $f(x) = x^3/4, \quad 0 < x < c;$
(b) $f(x) = (3/16)x^2, \quad -c < x < c;$
(c) $f(x) = c/\sqrt{x}, \quad 0 < x < 1$. Is this p.d.f. bounded?

1 **3.3-3.** For each of the following functions, (i) find the constant c so that $f(x)$ is a p.d.f. of a random variable X, (ii) find the distribution function, $F(x) = P(X \le x)$, and (iii) sketch graphs of the p.d.f. $f(x)$ and the distribution function $F(x)$:

(a) $f(x) = 4x^c, \quad 0 \le x \le 1,$
(b) $f(x) = c\sqrt{x}, \quad 0 \le x \le 4,$
(c) $f(x) = c/x^{3/4}, \quad 0 < x < 1.$

3.3-4. For each of the distributions in Exercise 3.3-2, find μ, σ^2, and σ.

2 **3.3-5.** For each of the distributions in Exercise 3.3-3, find μ, σ^2, and σ.

3.3-6. Let $f(x) = (1/2)x^2e^{-x}, 0 < x < \infty$, be the p.d.f. of X.

(a) Find the m.g.f. $M(t)$.
(b) Find the values of μ and σ^2.

3.3-7. Let $f(x) = (1/2)\sin x, 0 \le x \le \pi$, be the p.d.f. of X.

(a) Find μ and σ^2.
(b) Sketch the graph of the p.d.f. of X.
(c) Determine and sketch the graph of the distribution function of X.

3.3-8. The p.d.f. of X is $f(x) = c/x^2, 1 < x < \infty$.

(a) Calculate the value of c so that $f(x)$ is a p.d.f.
(b) Show that $E(X)$ is not finite.

2 **3.3-9.** The p.d.f. of Y is $g(y) = d/y^3, 1 < y < \infty$.

(a) Calculate the value of d so that $g(y)$ is a p.d.f.
(b) Find $E(Y)$. ✓
(c) Show that $\text{Var}(Y)$ is not finite. use defn of variance on page 135, multiply out and simplify, show then that the improper integral diverges

3.3-10. Sketch the graphs of the following probability density functions, and find and sketch the graphs of the distribution functions associated with these distributions (note carefully the relationship between the shape of the graph of the p.d.f. and the concavity of the graph of the distribution function):

(a) $f(x) = \left(\dfrac{3}{2}\right)x^2, \quad -1 < x < 1.$

(b) $f(x) = \dfrac{1}{2}, \quad -1 < x < 1.$

(c) $f(x) = \begin{cases} x + 1, & -1 < x < 0, \\ 1 - x, & 0 \le x < 1. \end{cases}$

3.3-11. Find the mean and variance for each of the distributions in Exercise 3.3-10.

3.3-12. Let $R(t) = \ln M(t)$, where $M(t)$ is the moment-generating function of a random variable. Show that

(a) $\mu = R'(0)$.

(b) $\sigma^2 = R''(0)$.

3.3-13. If $M(t) = e^{-t}(1 - t)^{-1}$, $t < 1$, use $R(t) = \ln M(t)$ and the result in Exercise 3.3-12 to find (a) μ and (b) σ^2.

3.3-14. Find the moment-generating function $M(t)$ of the distribution with p.d.f. $f(x) = (1/10)e^{-x/10}$, $0 < x < \infty$. Use $M(t)$ or $R(t) = \ln M(t)$ to determine the mean μ and the variance σ^2.

2 **3.3-15.** The logistic distribution is associated with the distribution function $F(x) = (1 + e^{-x})^{-1}$, $-\infty < x < \infty$. Find the p.d.f. of the logistic distribution and show that its graph is symmetric about $x = 0$.

Show $F'(-x) = F'(x)$ by multiplying numerator and denom by something appropriate

3.3-16. Let $f(x) = 1/2$, $0 < x < 1$ or $2 < x < 3$, zero elsewhere, be the p.d.f. of X.

(a) Sketch the graph of this p.d.f.

(b) Define the distribution function of X and sketch its graph.

(c) Find $q_1 = \pi_{0.25}$.

(d) Find $m = \pi_{0.50}$. Is it unique?

(e) Find $q_3 = \pi_{0.75}$.

1 **3.3-17.** The life X (in years) of a voltage regulator of a car has the p.d.f.

$$f(x) = \frac{3x^2}{7^3} e^{-(x/7)^3}, \qquad 0 < x < \infty.$$

(a) What is the probability that this regulator will last at least 7 years?

(b) Given that it has lasted at least 7 years, what is the conditional probability that it will last at least another 3.5 years?

1 **3.3-18.** Let $f(x) = (x + 1)/2$, $-1 < x < 1$. Find (a) $\pi_{0.64}$, (b) $q_1 = \pi_{0.25}$, and (c) $\pi_{0.81}$.

3.3-19. Let the random variable X_n have the p.d.f. $f(x_n) = n$, $0 < x_n < 1/n$.

(a) Define the distribution function of X_n—say, $F_n(x_n)$.

(b) Graph the p.d.f. and the distribution function for $n = 1, 5$, and 10.

3.3-20. It is interesting to note that back in 1940 if a person in the United States was selected at random, then his or her age X had a distribution that was almost given by the p.d.f. of the form

$$f(x) = \begin{cases} c, & 0 < x < 35, \\ c\left[1 + \dfrac{3}{140}(35 - x)\right], & 35 \le x \le 81\frac{2}{3}. \end{cases}$$

(a) Find c so that $f(x)$ is a p.d.f.

(b) In 1940, about what percentage of the U.S. population was older than 65?

(c) What was the median age then?

3.3-21. The lifetime X (in years) of a machine has a p.d.f.

$$f(x) = \frac{2x}{\theta^2} e^{-(x/\theta)^2}, \qquad 0 < x < \infty.$$

If $P(X > 5) = 0.01$, determine θ.

3.3-22. The weekly demand X for propane gas (in thousands of gallons) has the p.d.f.

$$f(x) = 4x^3 e^{-x^4}, \qquad 0 < x < \infty.$$

If the stockpile consists of two thousand gallons at the beginning of each week (and nothing extra is received during the week), what is the probability of not being able to meet the demand during a given week?

3.3-23. An insurance agent receives a bonus if the loss ratio L on his business is less than 0.5, where L is the total losses (say, X) divided by the total premiums (say, T). The bonus equals $(0.5 - L)(T/30)$ if $L < 0.5$ and equals zero otherwise. If X (in \$100,000) has the p.d.f.

$$f(x) = \frac{3}{x^4}, \qquad x > 1,$$

and if T (in \$100,000) equals 3, determine the expected value of the bonus.

3.3-24. The p.d.f. of time X to failure of an electronic component is

$$f(x) = \frac{2x}{1000^2} e^{-(x/1000)^2}, \qquad 0 < x < \infty.$$

(a) Compute $P(X > 2000)$.

(b) Determine the 75th percentile, $\pi_{0.75}$, of the distribution.

(c) Find the 10th and 60th percentiles, $\pi_{0.10}$ and $\pi_{0.60}$

3.3-25. The total amount of medical claims (in \$100,000) of the employees of a company has the p.d.f. $f(x) = 30x(1 - x)^4$, $0 < x < 1$. Find

(a) the mean and the standard deviation of the total in dollars,

(b) the probability that the total exceeds \$20,000.

3.3-26. Nicol (see references) lets the p.d.f. of X be defined by

$$f(x) = \begin{cases} x, & 0 \le x \le 1, \\ c/x^3, & 1 \le x < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

Find

(a) the value of c so that $f(x)$ is a p.d.f.
(b) the mean of X (if it exists).
(c) the variance of X (if it exists).
(d) $P(1/2 \le X \le 2)$.

EXERCISES

3.4-1. Show that the mean, variance, and moment-generating function of the uniform distribution are as given in this section.

3.4-2. Let $f(x) = 1/2$, $-1 \le x \le 1$, be the p.d.f. of X. Graph the p.d.f. and distribution function, and record the mean and variance of X.

3.4-3. Customers arrive randomly at a bank teller's window. Given that one customer arrived during a particular 10-minute period, let X equal the time within the 10 minutes that the customer arrived. If X is $U(0, 10)$, find

(a) the p.d.f. of X,

(b) $P(X \ge 8)$,

(c) $P(2 \le X < 8)$,

(d) $E(X)$, and

(e) $\text{Var}(X)$.

3.4-4. If the moment-generating function of X is

$$M(t) = \frac{e^{5t} - e^{4t}}{t}, \ t \neq 0, \qquad \text{and} \qquad M(0) = 1,$$

find (a) $E(X)$, (b) $\text{Var}(X)$, and (c) $P(4.2 < X \le 4.7)$.

3.4-5. Let Y have a uniform distribution $U(0,1)$, and let

$$W = a + (b - a)Y, \qquad a < b.$$

(a) Find the distribution function of W.
HINT: Find $P[a + (b - a)Y \le w]$.

(b) How is W distributed?

3.4-6. Let X have an exponential distribution with a mean of $\theta = 20$.

(a) Compute $P(10 < X < 30)$.

(b) Compute $P(X > 30)$.

(c) Compute $P(X > 40 \mid X > 10)$.

(d) What are the variance and the moment-generating function of X?

(e) The following ordered data were simulated from an exponential distribution with $\theta = 20$:

0.45	0.65	0.66	0.70	0.94	1.05	1.17	1.28	1.32	1.35
1.45	1.52	1.59	1.76	1.92	2.05	2.75	2.88	3.48	3.63
4.01	4.37	4.54	4.84	5.39	5.53	5.95	6.26	6.44	6.79
6.85	7.45	7.51	7.75	8.23	8.41	9.42	9.45	9.57	9.72
9.85	9.97	10.26	10.27	10.34	11.05	11.37	11.42	11.45	11.78
12.26	12.90	13.09	13.19	13.45	16.29	16.55	17.07	17.18	17.32
19.40	19.49	20.04	20.95	21.11	21.13	21.37	22.49	23.34	23.88
25.06	25.41	27.86	27.91	28.32	28.60	29.41	30.84	31.74	33.64
35.42	35.60	36.26	37.06	37.98	39.49	40.13	41.34	44.13	45.11
46.10	47.67	47.68	54.36	55.98	62.81	72.92	76.08	92.16	120.54

Compare the relative frequencies of the appropriate events in the data with the respective probabilities in parts (a), (b), and (c).

3.4-7. The *Holland Sentinel* reported the following numbers of calls per hour received by 911 between noon, February 26, and all day February 27:

3	0	3	4	9	1	6	2	2	5	7	6	4	2	2	4	1	0
3	1	3	3	4	2	1	3	3	2	3	2	5	2	1	0	2	4

The *Sentinel* also reported the following lengths of time per minute between calls:

30	17	65	8	38	35	4	19	7	14	12	4	5	4	2
7	5	12	50	33	10	15	2	10	1	5	30	41	21	31
1	18	12	5	24	7	6	31	1	3	2	22	1	30	2
1	3	12	12	9	28	6	50	63	5	17	11	23	2	46
90	13	21	55	43	5	19	47	24	4	6	27	4	6	37
16	41	68	9	5	28	42	3	42	8	52	2	11	41	4
35	21	3	17	10	16	1	68	105	45	23	5	10	12	17

(a) Show graphically that the numbers of calls per hour have an approximate Poisson distribution with a mean of $\lambda = 3$.

(b) Show that the sample mean and the sample standard deviation of the times between calls are both approximately equal to 20.

(c) If X is an exponential random variable with mean $\theta = 20$, compare the probability $P(X > 15)$ with the proportion of times that are greater than 15.

(d) Compare $P(X > 45.5 \mid X > 30.5)$ with the proportion of observations that satisfy this condition.

3.4-8. Telephone calls enter a college switchboard according to a Poisson process on the average of two every 3 minutes. Let X denote the waiting time until the first call that arrives after 10 A.M.

(a) What is the p.d.f. of X?

(b) Find $P(X > 2)$.

3.4-9. What are the p.d.f., the mean, and the variance of X if the moment-generating function of X is given by the following?

(a) $M(t) = \dfrac{1}{1 - 3t}, \quad t < 1/3.$

(b) $M(t) = \dfrac{3}{3 - t}, \quad t < 3.$

3.4-10. A biologist is studying the life cycle of the avian schistosome that causes swimmers itch. His study uses Menganser ducks for the adult parasites and aquatic snails as intermediate hosts for the larval stages. The life history is cyclic. (For more information, see http://swimmersitch.org/.) As a part of this study, the biologist and his students used snails from a natural population to measure the distances that snails travel. The conjecture is that snails that had a patent infection would not travel as far as those without such an infection.

Here are the measurements in cm that snails traveled per day. There are 39 in the infected group and 31 in the control group.

Distances for Infected Snail Group (ordered):

263	238	226	220	170	155	139	123	119	107	107	97	90
90	90	79	75	74	71	66	60	55	47	47	47	45
43	41	40	39	38	38	35	32	32	28	19	10	10

Distances for Control Snail Group (ordered):

314	300	274	246	190	186	185	182	180	141	132
129	110	100	95	95	93	83	55	52	50	48
48	44	40	32	30	25	24	18	7		

(a) Find the sample means and sample standard deviations for the two groups of snails.

(b) Make box plots of the two groups of snails on the same graph.

(c) Construct two q–q plots (y_r, π_p), where $p = r/(n+1)$ and $\pi_p = -\ln(1-p)$. That is, because we do not know the values of θ, use $\theta = 1$ in constructing the q–q plot.

(d) Do the data appear to be exponentially distributed?

(e) What are your conclusions about the snails?

3.4-11. Let X have an exponential distribution with mean $\theta > 0$. Show that

$$P(X > x + y \mid X > x) = P(X > y).$$

3.4-12. Let $F(x)$ be the distribution function of the continuous-type random variable X, and assume that $F(x) = 0$ for $x \le 0$ and $0 < F(x) < 1$ for $0 < x$. Prove that if

$$P(X > x + y \mid X > x) = P(X > y),$$

then

$$F(x) = 1 - e^{-\lambda x}, \quad 0 < x.$$

HINT: Show that $g(x) = 1 - F(x)$ satisfies the functional equation

$$g(x + y) = g(x)g(y),$$

which implies that $g(x) = a^{cx}$.

3.4-13. Let X equal the number of bad records in each 100 feet of a used computer tape. Assume that X has a Poisson distribution with mean 2.5. Let W equal the number of feet before the first bad record is found.

(a) Give the mean number of flaws per foot.

(b) How is W distributed?

(c) Give the mean and variance of W.

(d) Find (i) $P(W \leq 20)$, (ii) $P(W > 40)$, and (iii) $P(W > 60 \mid W > 20)$.

3.4-14. The initial value of an appliance is \$700 and its value in the future is given by

$$v(t) = 100\,(2^{3-t} - 1), \qquad 0 \leq t \leq 3,$$

where t is time in years. Thus, after the first 3 years the appliance is worth nothing as far as the warranty is concerned. If it fails in the first three years, the warranty pays $v(t)$. Compute the expected value of the payment on the warranty if T has an exponential distribution with mean five.

3.4-15. Let X equal the time (in minutes) between calls that are made over the public safety radio. On four different days (February 14, 21, and 28, and March 6) and during a period of one hour on each day, the following observations of X were made:

5	7	8	20	17	2	24	8	8	6	4
3	42	10	18	5	7	8	4	5	10	

If calls arrive randomly in accordance with an approximate Poisson process, then the distribution of X should be approximately exponential.

(a) Calculate the values of the sample mean and sample standard deviation. Are they close to each other in value?

(b) Construct a q–q plot of the ordered observations versus the respective quartiles of the exponential distribution with a mean of $\theta = 1$. If this plot is approximately linear, the exponential model is supported. Since the mean of these data is not close to 1, the line plotted will not have slope 1, but a linear fit will still indicate an exponential model. What is your conclusion?

3.4-16. A grocery store has n watermelons to sell and makes \$1.00 on each sale. Say the number of consumers of these watermelons is a random variable with a distribution that can be approximated by

$$f(x) = \frac{1}{200}, \qquad 0 < x < 200,$$

a p.d.f. of the continuous type. If the grocer does not have enough watermelons to sell to all consumers, she figures that she loses \$5.00 in goodwill from each unhappy customer. But if she has surplus watermelons, she loses 50 cents on each extra watermelon. What should n be to maximize profit? HINT: If $X \leq n$, then her profit is $(1.00)X + (-0.50)(n - X)$; but if $X > n$, her profit is $(1.00)n + (-5.00)(X - n)$. Find the expected value of profit as a function of n, and then select n to maximize that function.

3.4-17. There are times when a shifted exponential model is appropriate. That is, let the p.d.f. of X be

$$f(x) = \frac{1}{\theta}e^{-(x-\delta)/\theta}, \qquad \delta < x < \infty.$$

(a) Define the distribution function of X.

(b) Calculate the mean and variance of X.

3.4-18. A certain type of aluminum screen 2 feet in width has, on the average, three flaws in a 100-foot roll.

(a) What is the probability that the first 40 feet in a roll contain no flaws?

(b) What assumption did you make to solve part (a)?

3.4-19. Let X have an exponential distribution with mean θ.

(a) Find the first quartile, q_1.

(b) How far is the first quartile below the mean?

(c) Find the third quartile, q_3.

(d) How far is the third quartile above the mean?

3.4-20. Let X have a logistic distribution with p.d.f.

$$f(x) = \frac{e^{-x}}{(1 + e^{-x})^2}, \qquad -\infty < x < \infty.$$

Show that

$$Y = \frac{1}{1 + e^{-X}}$$

has a $U(0,1)$ distribution. HINT: Find $G(y) = P(Y \leq y) = P\left(\frac{1}{1 + e^{-X}} \leq y\right)$ when $0 < y < 1$.

3.4-21. Suppose that the length of life of a human female, X, is modeled by the exponential p.d.f.

$$f(x) = \frac{1}{80}e^{-x/80}, \qquad 0 < x < \infty.$$

(a) Compute the probability $P(X > 10)$ and also compute the conditional probability $P(X > 90 \mid X > 80)$. Note that the answers are the same, indicating that if length of life has an exponential distribution, then those following it would have a "mathematical fountain of youth." Unfortunately, this is **not** a good model of length of life.

(b) Possibly a more realistic model for length of life is the distribution function

$$F(x) = 1 - \exp[-(a/b)(e^{bx} - 1)], \quad 0 < x < \infty, \text{ where } a > 0, b > 0.$$

Find and graph $f(x) = F'(x)$

(c) With $a = 0.000025$ and $b = 0.1$, determine the percentile of $x = 70$.

3.4-22. Let the random variable X be equal to the number of days that it takes a high-risk driver to have an accident. Assume that X has an exponential distribution. If $P(X < 50) = 0.25$, compute $P(X > 100 \mid X > 50)$.

3.4-23. A loss (in \$100,000) due to fire in a building has a p.d.f. $f(x) = (1/6)e^{-x/6}$, $0 < x < \infty$. Given that the loss is greater than 5, find the probability that it is greater than 8.

EXERCISES

3.5-1. Telephone calls enter a college switchboard at a mean rate of two-thirds of a call per minute according to a Poisson process. Let X denote the waiting time until the tenth call arrives.

(a) What is the p.d.f. of X?

(b) What are the moment-generating function, mean, and variance of X?

3.5-2. If X has a gamma distribution with $\theta = 4$ and $\alpha = 2$, find $P(X < 5)$.

3.5-3. Find the moment-generating function for the gamma distribution with parameters α and θ.

HINT: In the integral representing $E(e^{tX})$, change variables by letting $y = (1 - \theta t)x/\theta$, where $1 - \theta t > 0$.

3.5-4. Use the moment-generating function of a gamma distribution to show that $E(X) = \alpha\theta$ and $\mathrm{Var}(X) = \alpha\theta^2$.

3.5-5. If the moment-generating function of a random variable W is

$$M(t) = (1 - 7t)^{-20},$$

find the p.d.f., mean, and variance of W.

3.5-6. Let X denote the number of alpha particles emitted by barium-133 and observed by a Geiger counter in a fixed position. Assume that X has a Poisson distribution and $\lambda = 14.7$ is the mean number of counts per second. Let W denote the waiting time to observe 100 counts. Twenty-five independent observations of W are

6.9	7.3	6.7	6.4	6.3
5.9	7.0	7.1	6.5	7.6
7.2	7.1	6.1	7.3	7.6
7.6	6.7	6.3	5.7	6.7
7.5	5.3	5.4	7.4	6.9

(a) Give the p.d.f., mean, and variance of W.

(b) Calculate the sample mean and sample variance of the 25 observations of W.

(c) Use the relative frequency of event $\{W \le 6.6\}$ to approximate $P(W \le 6.6)$.

3.5-7. The waiting times in minutes until two calls to 911, as reported by the *Holland Sentinel* on November 13 between noon and midnight, were

20	28	81	4	9	41	9	11	10	24	20
44	18	30	16	53	15	38	50	84	44	69

Could these times represent a random sample from a gamma distribution with $\alpha = 2$ and $\theta = 120/7$?

(a) Compare the distribution and sample means.
(b) Compare the distribution and sample variances.
(c) Compare $P(X < 35)$ with the proportion of times that are less than 35.
(d) If possible, make some graphical comparisons. For example, if you have access to Minitab, construct a q–q plot for these data.
(e) What is your conclusion?

3.5-8. Let X equal the number of alpha particle emissions of carbon-14 that are counted by a Geiger counter each second. Assume that the distribution of X is Poisson with mean 16. Let W equal the time in seconds before the seventh count is made.

(a) Give the distribution of W.
(b) Find $P(W \leq 0.5)$. HINT: Use Equation 3.5-1 with $\lambda w = 8$.

3.5-9. If X is $\chi^2(17)$, find

(a) $P(X < 7.564)$.
(b) $P(X > 27.59)$.
(c) $P(6.408 < X < 27.59)$.
(d) $\chi^2_{0.95}(17)$.
(e) $\chi^2_{0.025}(17)$.

3.5-10. If X is $\chi^2(12)$, find constants a and b such that

$$P(a < X < b) = 0.90 \text{ and } P(X < a) = 0.05.$$

3.5-11. If X is $\chi^2(23)$, find the following:

(a) $P(14.85 < X < 32.01)$.
(b) Constants a and b such that $P(a < X < b) = 0.95$ and $P(X < a) = 0.025$.
(c) The mean and variance of X.
(d) $\chi^2_{0.05}(23)$ and $\chi^2_{0.95}(23)$.

3.5-12. If the moment-generating function of X is $M(t) = (1 - 2t)^{-12}, t < 1/2$, find

(a) $E(X)$.
(b) $\text{Var}(X)$.
(c) $P(15.66 < X < 42.98)$.

3.5-13. Let the distribution of X be $\chi^2(r)$.

(a) Find the point at which the p.d.f. of X attains its maximum when $r \geq 2$. This is the mode of a $\chi^2(r)$ distribution.
(b) Find the points of inflection for the p.d.f. of X.
(c) Use the results of parts (a) and (b) to sketch the p.d.f. of X when $r = 4$ and when $r = 10$.

3.5-14. Cars arrive at a toll booth at a mean rate of five cars every 10 minutes according to a Poisson process. Find the probability that the toll collector will have to wait longer than 26.30 minutes before collecting the eighth toll.

3.5-15. If 15 observations are taken independently from a chi-square distribution with four degrees of freedom, find the probability that at most 3 of the 15 observations exceed 7.779.

3.5-16. If 10 observations are taken independently from a chi-square distribution with 19 degrees of freedom, find the probability that exactly 2 of the 10 sample items exceed 30.14.

3.5-17. In a medical experiment, a rat has been exposed to some radiation. The experimenters believe that the rat's survival time X (in weeks) has the p.d.f.

$$f(x) = \frac{3x^2}{120^3} e^{-(x/120)^3}, \qquad 0 < x < \infty.$$

(a) What is the probability that the rat survives at least 100 weeks?
(b) Find the expected value of the survival time. HINT: In the integral representing $E(X)$, let $y = (x/120)^3$ and get the answer in terms of a gamma function.

3.5-18. Say the serum cholesterol level (X) of U.S. males ages 25–34 follows a translated gamma distribution with p.d.f.

$$f(x) = \frac{x - 80}{50^2} e^{-(x - 80)/50}, \quad 80 < x < \infty.$$

(a) What are the mean and the variance of this distribution?
(b) What is the mode?
(c) What percentage have the model cholesterol level less than 200? HINT: Integrate by parts.

3.5-19. A bakery sells rolls in units of a dozen. The demand X (in 1000 units) for rolls has a gamma distribution with parameters $\alpha = 3, \theta = 0.5$. It costs 40 cents to make a unit that sells for \$1 on the first day when the rolls are fresh. Any leftover units are sold on the second day at 20 cents. How many units should be made to maximize the expected value of the profit?

EXERCISES

3.6-1. If Z is $N(0,1)$, find

(a) $P(0.53 < Z \le 2.06)$.
(b) $P(-0.79 \le Z < 1.52)$.
(c) $P(Z > -1.77)$.
(d) $P(Z > 2.89)$.
(e) $P(|Z| < 1.96)$.
(f) $P(|Z| < 1)$.
(g) $P(|Z| < 2)$.
(h) $P(|Z| < 3)$.

3.6-2. If Z is $N(0,1)$, find

(a) $P(0 \le Z \le 0.87)$.
(b) $P(-2.64 \le Z \le 0)$.
(c) $P(-2.13 \le Z \le -0.56)$.
(d) $P(|Z| > 1.39)$.
(e) $P(Z < -1.62)$.
(f) $P(|Z| > 1)$.
(g) $P(|Z| > 2)$.
(h) $P(|Z| > 3)$.

3.6-3. Find the values of (a) $z_{0.01}$, (b) $-z_{0.005}$, (c) $z_{0.0475}$, and (d) $z_{0.985}$.

3.6-4. Find the values of (a) $z_{0.10}$, (b) $-z_{0.05}$, (c) $-z_{0.0485}$, and (d) $z_{0.9656}$.

3.6-5. If Z is $N(0,1)$, find values of c such that

(a) $P(Z \ge c) = 0.025$.
(b) $P(|Z| \le c) = 0.95$.
(c) $P(Z > c) = 0.05$.
(d) $P(|Z| \le c) = 0.90$.

3.6-6. If the moment-generating function of X is $M(t) = \exp(166t + 200t^2)$, find

(a) The mean of X.
(b) The variance of X.
(c) $P(170 < X < 200)$.
(d) $P(148 \le X \le 172)$.

3.6-7. If X is normally distributed with a mean of 6 and a variance of 25, find

(a) $P(6 \le X \le 12)$.
(b) $P(0 \le X \le 8)$.
(c) $P(-2 < X \le 0)$.
(d) $P(X > 21)$.
(e) $P(|X - 6| < 5)$.
(f) $P(|X - 6| < 10)$.
(g) $P(|X - 6| < 15)$.
(h) $P(|X - 6| < 12.41)$.

3.6-8. Let the distribution of X be $N(\mu, \sigma^2)$ Show that the points of inflection of the graph of the p.d.f. of X occur at $x = \mu \pm \sigma$.

3.6-9. If X is $N(650, 625)$, find

(a) $P(600 \le X < 660)$.

(b) A constant $c > 0$ such that $P(|X - 650| \le c) = 0.9544$.

read

3.6-10. If X is $N(\mu, \sigma^2)$, show that $Y = aX + b$ is $N(a\mu + b, a^2\sigma^2)$, $a \ne 0$. Hint: Find the distribution function $P(Y \le y)$ of Y, and in the resulting integral, let $w = ax + b$ or, equivalently, $x = (w - b)/a$.

3.6-11. Find the distribution of $W = X^2$ when

(a) X is $N(0, 4)$,

(b) X is $N(0, \sigma^2)$.

3.6-12. A company manufactures windows that are inserted into an automobile. Each window has five studs for attaching it. A pullout test is used to determine the force required to pull a stud out of a window. (Note that this is an example of destructive testing.) Let X equal the force required for pulling studs out of position 4. Sixty observations of X were as follows:

159	150	147	160	155	142	143	151	154	133
151	146	140	146	137	148	154	157	142	153
135	144	135	165	118	158	126	147	123	140
125	151	153	158	144	163	150	150	137	164
137	156	139	134	171	144	160	147	155	175
162	160	149	149	158	152	165	131	150	120

(a) Construct an ordered stem-and-leaf diagram, using 11•, 12*, 12•, and so on as stems.

(b) Construct a q–q plot, using the ordered array and the corresponding quantiles of $N(0,1)$. (You will probably want to use a statistical package on the computer to help you out.)

(c) Does it look like X has a normal distribution?

3.6-13. Some measurements (in mm) were made on specimens of the spider Sosippus floridanus, which is native to Florida. Here are the lengths of 9 female spiders and 9 male spiders.

Female spiders	11.06	13.87	12.93	15.08	17.82
	14.14	12.26	17.82	20.17	
Male spiders	12.26	11.66	12.53	13.00	11.79
	12.46	10.65	10.39	12.26	

(a) Construct a q–q plot, $N(0,1)$ quantiles versus the ordered female spider lengths. Do they appear to be normally distributed?

(b) Construct a q–q plot, $N(0,1)$ quantiles versus the ordered male spider lengths. Do they appear to be normally distributed?

3.6-14. A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints is $N(21.37, 0.16)$.

(a) Let X denote the weight of a single mint selected at random from the production line. Find $P(X > 22.07)$.

(b) Suppose that 15 mints are selected independently and weighed. Let Y equal the number of these mints that weigh less than 20.857 grams. Find $P(Y \le 2)$.

3.6-15. A chemistry major weighed 19 plain M&M's (in grams) on a ± 0.0001 scale. The ordered weights are as follows:

0.7938	0.8032	0.8089	0.8222	0.8268	0.8383	0.8442
0.8490	0.8528	0.8572	0.8674	0.8734	0.8786	
0.8850	0.8873	0.8920	0.9069	0.9150	0.9243	

(a) Construct a stem-and-leaf diagram using three-digit leaves, with stems 0.7•, 0.8∗, 0.8t, and so on.

(b) Construct a q-q plot of these data and the corresponding quantiles for the standard normal distribution, $N(0,1)$.

(c) Do the data look like observations from a normal distribution? Why?

3.6-16. If the moment-generating function of X is $M(t) = e^{500t + 5000t^2}$, find $P[27{,}060 \le (X - 500)^2 \le 50{,}240]$.

3.6-17. If X is $N(7, 4)$, find $P[15.364 \le (X - 7)^2 \le 20.096]$.

3.6-18. The strength X of a certain material is such that its distribution is found by $X = e^Y$, where Y is $N(10, 1)$. Find the distribution function and p.d.f. of X, and compute $P(10{,}000 < X < 20{,}000)$. Note: The random variable X is said to have a **lognormal distribution.**

3.6-19. A packaged product has a label weight of 450 grams. The company goal is to fill each package with at least 450 grams, but at most 458 grams. To check these goals, a random sample of 100 packages was selected and weighed, yielding the following weights, rounded to the nearest gram:

457	457	455	457	454	454	457	455	456	459
457	458	456	456	461	457	458	452	457	460
453	458	452	454	454	456	455	456	451	454
456	457	457	453	455	459	458	457	458	457
461	457	455	458	458	455	457	458	456	463
455	455	455	456	456	456	455	456	460	456
456	457	458	454	455	456	459	457	457	451
450	453	453	459	450	453	452	458	456	457
451	458	456	460	455	455	456	460	457	456
457	456	460	459	457	455	461	455	457	457

(a) Construct a frequency table and a corresponding relative frequency histogram.

(b) Do these data seem to come from a normal distribution with mean μ about equal to $\overline{x} = 456.2$ and variance about equal to $s^2 = 5.96$? Sketch the p.d.f. for the normal distribution $N(456.2, 5.96)$ on your histogram.

3.6-20. Nine measurements are taken on the strength of a certain metal. In order, they are 7.2, 8.9, 9.7, 10.5, 10.9, 11.7, 12.9, 13.9, 15.3, and these values correspond to the 10th, 20th, $\cdots$, 90th percentiles of this sample. Plot the measurements against the same percentiles of $N(0, 1)$. Does it seem reasonable that the underlying distribution of strengths could be normal?

3.6-21. The graphs of the moment-generating functions of three normal distributions—$N(0,1)$, $N(-1,1)$, and $N(2,1)$—are given in Figure 3.6-4(a). Identify them.

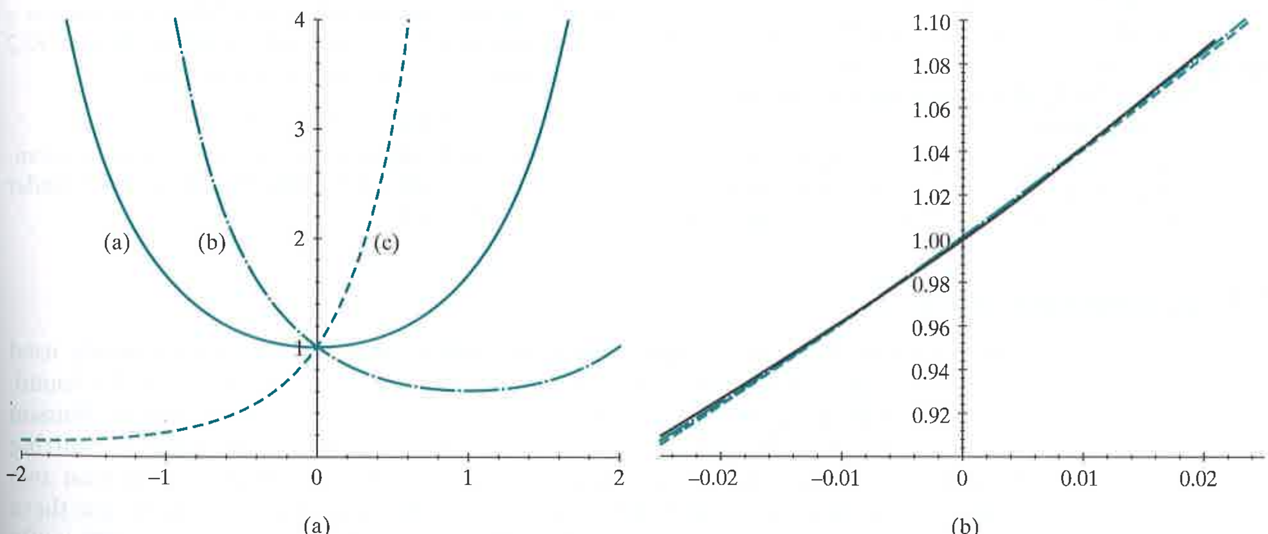

FIGURE 3.6-4: Moment-generating functions

3.6-22. Figure 3.6-4(b) shows the graphs of the following three moment-generating functions:

$$g_1(t) = \frac{1}{1 - 4t}, \qquad t < 1/4,$$

$$g_2(t) = \frac{1}{(1 - 2t)^2}, \qquad t < 1/2,$$

$$g_3(t) = e^{4t+t^2/2}.$$

Why do these three graphs look so similar around $t = 0$?

3.6-23. The serum zinc level X in micrograms per deciliter for males between ages 15 and 17 has a distribution that is approximately normal with $\mu = 90$ and $\sigma = 15$. Compute the conditional probability $P(X > 120 \mid X > 105)$.

3.6-24. An interior automotive supplier places several electrical wires in a harness. A pull test measures the force required to pull spliced wires apart. A customer requires that each wire that is spliced into the harness withstand a pull force of 20 pounds. Let X equal the force required to pull the spliced wire apart. The following data give the values of a random sample of $n = 20$ observations of X:

28.8	24.4	30.1	25.6	26.4	23.9	22.1	22.5	27.6	28.1
20.8	27.7	24.4	25.1	24.6	26.3	28.2	22.2	26.3	24.4

(a) Construct a q–q plot, using the ordered array and the corresponding quantiles of $N(0, 1)$.

(b) Does it look like X has a normal distribution?

3.6-25. The "fill" problem is important in many industries, such as those making cereal, toothpaste, beer, and so on. If an industry claims that it is selling 12 ounces of its product in a container, it must have a mean greater than 12 ounces, or else the FDA will crack down, although the FDA will allow a very small percentage of the containers to have less than 12 ounces.

(a) If the content X of a container has a $N(12.1, \sigma^2)$ distribution, find σ so that $P(X < 12) = 0.01$.

(b) If $\sigma = 0.05$, find μ so that $P(X < 12) = 0.01$.

3.6-26. The ordered weights (in grams) of 24 bags of peanut M & M's, with a label weight of 49.2 g, are as follows:

52.3	53.6	54.0	54.1	54.4	54.5	54.7	55.2
55.4	55.4	55.5	55.7	55.9	56.1	56.5	56.5
57.0	57.0	57.3	57.3	57.3	57.8	59.6	59.7

(a) Calculate the sample mean and sample standard deviation of these weights.

(b) Construct a q–q plot, using the ordered weights and the percentiles of a standard normal distribution.

(c) Do these data look like they come from a normal distribution?

(d) Are the fill weights appropriate for this label weight?

3.6-27. Assume that the fill X of a filling machine for a beverage has a normal distribution with $\mu = 12.2$ and $\sigma = 0.1$, measured in fluid ounces.

(a) Compute $P(X < 12)$.

(b) In 50 independent such measurements, compute the probability that at least 1 is under 12 ounces.