puter to perform the simulation; or simply use the table of random numbers (Table IX in the appendix), start at a random spot, and let an integer in the set $\{0,1\} = 1$, in $\{2,3,4\} = 2$, in $\{5,6,7\} = 3$, in $\{8,9\} = 4$.

(c) Plot $f(x)$ and $h(x)$ on the same graph.

1.1-9. Toss two coins at random and count the number of heads that appear "up." Here $S = \{0,1,2\}$. In Chapter 2, we discover that a reasonable probability model is given by the p.m.f. $f(0) = 1/4, f(1) = 1/2, f(2) = 1/4$. Repeat this experiment at least $n = 100$ times, and plot the resulting relative frequency histogram $h(x)$ on the same graph with $f(x)$.

1.1-10. Let the random variable X be the number of tosses of a coin needed to obtain the first head. Here $S = \{1,2,3,4,\ldots\}$. In Chapter 2, we find that a reasonable probability model is given by the p.m.f. $f(x) = (1/2)^x$, $x \in S$. Do this experiment a large number of times, and compare the resulting relative frequency histogram $h(x)$ with $f(x)$.

1.1-11. In 1985, Al Bumbry of the Baltimore Orioles and Darrell Brown of the Minnesota Twins had the following numbers of hits (H) and official at bats (AB) on grass and artificial turf:

Playing Surface	Bumbry			Brown		
	AB	H	BA	AB	H	BA
Grass	295	77		92	18	
Artificial Turf	49	16		168	53	
Total	344	93		260	71	

(a) Find the batting averages BA (namely, H/AB) of each player on grass.
(b) Find the BA of each player on artificial turf.
(c) Find the season batting averages for the two players.
(d) Interpret your results.

1.1-12. In 1985, Kent Hrbek of the Minnesota Twins and Dion James of the Milwaukee Brewers had the following numbers of hits (H) and official at bats (AB) on grass and artificial turf:

Playing Surface	Hrbek			James		
	AB	H	BA	AB	H	BA
Grass	204	50		329	93	
Artificial Turf	355	124		58	21	
Total	559	174		387	114	

(a) Find the batting averages BA (namely, H/AB) of each player on grass.
(b) Find the BA of each player on artificial turf.
(c) Find the season batting averages for the two players.
(d) Interpret your results.

1.1-13. If we had a choice of two airlines, we would possibly choose the airline with the better "on-time performance." So consider Alaska Airlines and America West, using data reported by Arnold Barnett (see references):

Airline	Alaska Airlines	America West
Destination	Relative Frequency on Time	Relative Frequency on Time
Los Angeles	$\dfrac{497}{559}$	$\dfrac{694}{811}$
Phoenix	$\dfrac{221}{233}$	$\dfrac{4840}{5255}$
San Diego	$\dfrac{212}{232}$	$\dfrac{383}{448}$
San Francisco	$\dfrac{503}{605}$	$\dfrac{320}{449}$
Seattle	$\dfrac{1841}{2146}$	$\dfrac{201}{262}$
Five-City Total	$\dfrac{3274}{3775}$	$\dfrac{6438}{7225}$

(a) For each of the five cities listed, which airline has the better on-time performance?
(b) Combining the results, which airline has the better on-time performance?
(c) Interpret your results.

1.2 PROPERTIES OF PROBABILITY

In Section 1.1, the collection of all possible outcomes (the **universal set**) of a random experiment is denoted by S and is called the **outcome space**. Given an outcome space S, let A be a part of the collection of outcomes in S; that is, $A \subset S$. Then A is called an **event**. When the random experiment is performed and the outcome of the experiment is in A, we say that **event A has occurred**.

Since, in studying probability, the words *set* and *event* are interchangeable, the reader might want to review **algebra of sets**, found in Appendix D on the CD-ROM. For convenience, however, here we remind the reader of some terminology:

- $\varnothing$ denotes the **null** or **empty** set;
- $A \subset B$ means A is a **subset** of B;
- $A \cup B$ is the **union** of A and B;
- $A \cap B$ is the **intersection** of A and B;
- A' is the **complement** of A (i.e., all elements in S that are not in A).

Some of these sets are depicted by the shaded region in Figure 1.2-1, in which S is the interior of the rectangles. Such figures are called **Venn diagrams**.
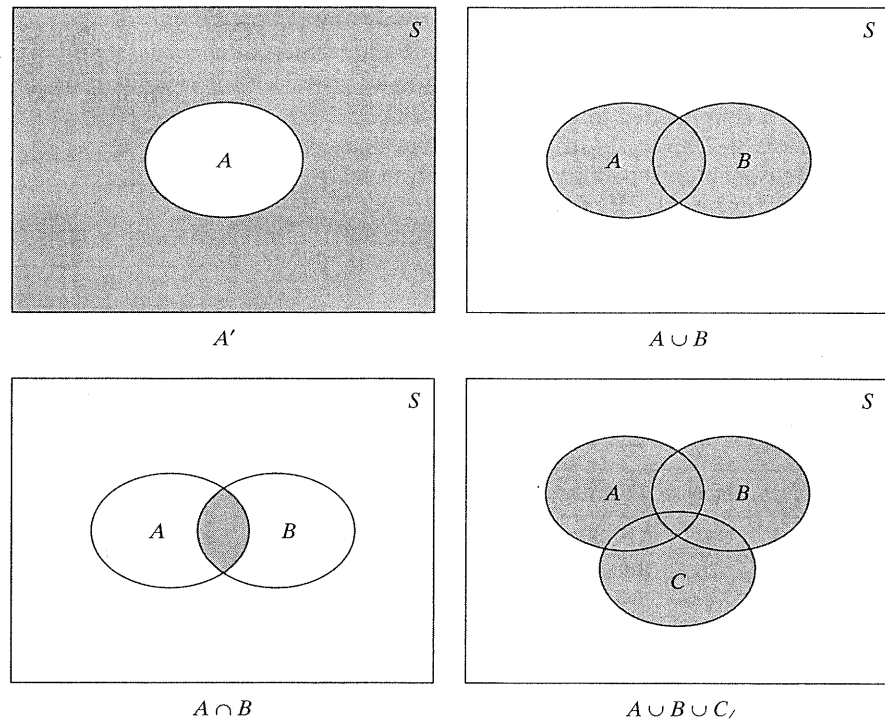

FIGURE 1.2-1: Algebra of sets

Special terminology associated with events that is often used by statisticians includes the following:

1. $A_1, A_2, \ldots, A_k$ are **mutually exclusive events** means that $A_i \cap A_j = \varnothing, i \neq j$, that is, $A_1, A_2, \ldots, A_k$ are disjoint sets;

2. $A_1, A_2, \ldots, A_k$ are **exhaustive events** means that $A_1 \cup A_2 \cup \cdots \cup A_k = S$.

So if $A_1, A_2, \ldots, A_k$ are **mutually exclusive and exhaustive** events, we know that $A_i \cap A_j = \varnothing, i \neq j$, and $A_1 \cup A_2 \cup \cdots \cup A_k = S$.

We are interested in defining what is meant by the probability of event A, denoted by $P(A)$ and often called the chance of A occurring. To help us understand what is meant by the probability of A, consider repeating the experiment a number of times—say, n times. Count the number of times that event A actually occurred throughout these n performances; this number is called the frequency of event A and is denoted by $\mathcal{N}(A)$. The ratio $\mathcal{N}(A)/n$ is called the **relative frequency** of event A

in these n repetitions of the experiment. A relative frequency is usually very unstable for small values of n, but it tends to stabilize as n increases. This suggests that we associate with event A a number—say, p—that is equal to or approximately equal to the number about which the relative frequency tends to stabilize. This number p can then be taken as the number that the relative frequency of event A will be near in future performances of the experiment. Thus, although we cannot predict the outcome of a random experiment with certainty, we can, for a large value of n, predict fairly accurately the relative frequency associated with event A. The number p assigned to event A is called the **probability** of event A and is denoted by $P(A)$. That is, $P(A)$ represents the proportion of outcomes of a random experiment that terminate in the event A as the number of trials of that experiment increases without bound.

The next example will help to illustrate some of the ideas just presented.

EXAMPLE 1.2-1 A fair six-sided die is rolled six times. If the face numbered k is the outcome on roll k for $k = 1, 2, \ldots, 6$, we say that a match has occurred. The experiment is called a success if at least one match occurs during the six trials. Otherwise, the experiment is called a failure. The sample space is $S = \{\text{success, failure}\}$. Let $A = \{\text{success}\}$. We would like to assign a value to $P(A)$. Accordingly, this experiment was simulated 500 times on a computer. Figure 1.2-2 depicts the results of this simulation, and the following table summarizes a few of the results:

n	$\mathcal{N}(A)$	$\mathcal{N}(A)/n$
50	37	0.740
100	69	0.690
250	172	0.688
500	330	0.660

The probability of event A is not intuitively obvious, but it will be shown in Example 1.5-6 that $P(A) = 1 - (1 - 1/6)^6 = 0.665$. This assignment is certainly supported by the simulation (although not proved by it). ∎
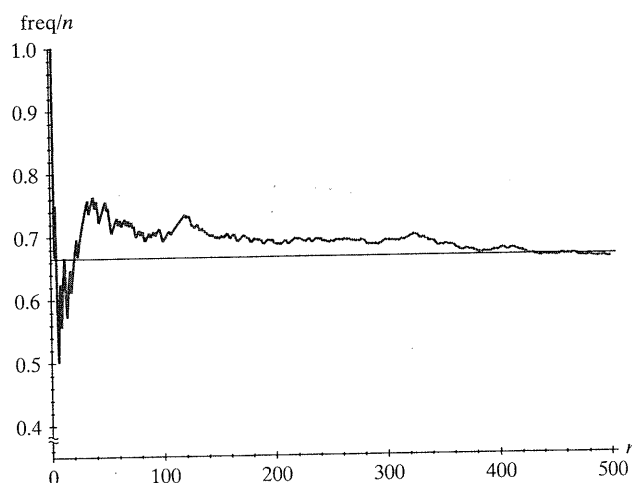

FIGURE 1.2-2: Fraction of experiments having at least one match

Example 1.2-1 shows that at times intuition cannot be used to assign probabilities, although simulation can perhaps help to assign a probability empirically. The next example illustrates where intuition can help in assigning a probability to an event.

EXAMPLE 1.2-2 A disk 2 inches in diameter is thrown at random on a tiled floor, where each tile is a square with sides 4 inches in length. Let C be the event that the disk will land entirely on one tile. In order to assign a value to $P(C)$, consider the center of the disk. In what region must the center lie to ensure that the disk lies entirely on one tile? If you draw a picture, it should be clear that the center must lie within a square having sides of length 2 and with its center coincident with the center of a tile. Since the area of this square is 4 and the area of a tile is 16, it makes sense to let $P(C) = 4/16$. ∎

Sometimes the nature of an experiment is such that the probability of A can be assigned easily. For example, when a state lottery randomly selects a three-digit integer, we would expect each of the 1000 possible three-digit numbers to have the same chance of being selected, namely, $1/1000$. If we let $A = \{233, 323, 332\}$, then it makes sense to let $P(A) = 3/1000$. Or if we let $B = \{234, 243, 324, 342, 423, 432\}$, then we would let $P(B) = 6/1000$, the probability of the event B. Probabilities of events associated with many random experiments are perhaps not quite as obvious and straightforward as was seen in Example 1.2-1.

So we wish to associate with A a number $P(A)$ about which the relative frequency $\mathcal{N}(A)/n$ of the event A tends to stabilize with large n. A function such as $P(A)$ that is evaluated for a set A is called a **set function**. In this section, we consider the probability set function $P(A)$ and discuss some of its properties. In succeeding sections, we shall describe how the probability set function is defined for particular experiments.

To help decide what properties the probability set function should satisfy, consider properties possessed by the relative frequency $\mathcal{N}(A)/n$. For example, $\mathcal{N}(A)/n$ is always nonnegative. If $A = S$, the sample space, then the outcome of the experiment will always belong to S, and thus $\mathcal{N}(S)/n = 1$. Also, if A and B are two mutually exclusive events, then $\mathcal{N}(A \cup B)/n = \mathcal{N}(A)/n + \mathcal{N}(B)/n$. Hopefully, these remarks will help to motivate the following definition.

DEFINITION 1.2-1

Probability is a real-valued set function P that assigns, to each event A in the sample space S, a number $P(A)$, called the probability of the event A, such that the following properties are satisfied:

(a) $P(A) \geq 0$,

(b) $P(S) = 1$,

(c) If $A_1, A_2, A_3, \ldots$ are events and $A_i \cap A_j = \emptyset$, $i \neq j$, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$$

for each positive integer k, and

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

for an infinite, but countable, number of events.

The theorems that follow give some other important properties of the probability set function. When one considers these theorems, it is important to understand the

theoretical concepts and proofs. However, if the reader keeps the relative frequency concept in mind, the theorems should also have some intuitive appeal.

THEOREM 1.2-1

For each event A,

$$P(A) = 1 - P(A').$$

Proof. We have

$$S = A \cup A' \quad \text{and} \quad A \cap A' = \emptyset.$$

Thus, from properties (b) and (c), it follows that

$$1 = P(A) + P(A').$$

Hence,

$$P(A) = 1 - P(A'). \qquad \square$$

EXAMPLE 1.2-3

A fair coin is flipped successively until the same face is observed on successive flips. Let $A = \{x : x = 3, 4, 5, \ldots\}$; that is, A is the event that it will take three or more flips of the coin to observe the same face on two consecutive flips. To find $P(A)$, we first find the probability of $A' = \{x : x = 2\}$, the complement of A. In two flips of a coin, the possible outcomes are $\{HH, HT, TH, TT\}$, and we assume that each of these four points has the same chance of being observed. Thus,

$$P(A') = P(\{HH, TT\}) = \frac{2}{4}.$$

It follows from Theorem 1.2-1 that

$$P(A) = 1 - P(A') = 1 - \frac{2}{4} = \frac{2}{4}. \qquad \blacksquare$$

THEOREM 1.2-2

$P(\emptyset) = 0.$

Proof. In Theorem 1.2-1, take $A = \emptyset$ so that $A' = S$. Then

$$P(\emptyset) = 1 - P(S) = 1 - 1 = 0. \qquad \square$$

THEOREM 1.2-3

If events A and B are such that $A \subset B$, then $P(A) \leq P(B)$.

Proof. We have

$$B = A \cup (B \cap A') \quad \text{and} \quad A \cap (B \cap A') = \emptyset.$$

Hence, from property (c),

$$P(B) = P(A) + P(B \cap A') \geq P(A)$$

because, from property (a),

$$P(B \cap A') \geq 0.$$

<div>

THEOREM 1.2-4

For each event A, $P(A) \leq 1$.

</div>

Proof. Since $A \subset S$, we have, by Theorem 1.2-3 and property (b),

$$P(A) \leq P(S) = 1,$$

which gives the desired result.

Property (a), along with Theorem 1.2-4, shows that, for each event A,

$$0 \leq P(A) \leq 1.$$

<div>

THEOREM 1.2-5

If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

</div>

Proof. The event $A \cup B$ can be represented as a union of mutually exclusive events, namely,

$$A \cup B = A \cup (A' \cap B).$$

Hence, by property (c),

$$P(A \cup B) = P(A) + P(A' \cap B). \qquad (1.2\text{-}1)$$

However,

$$B = (A \cap B) \cup (A' \cap B),$$

which is a union of mutually exclusive events. Thus,

$$P(B) = P(A \cap B) + P(A' \cap B)$$

and

$$P(A' \cap B) = P(B) - P(A \cap B).$$

If the right-hand side of this equation is substituted into Equation 1.2-1, we obtain

$$P(A \cup B) = P(A) + P(B) - P(A \cap B),$$

which is the desired result.

EXAMPLE 1.2-4 A faculty leader was meeting two students in Paris, one arriving by train from Amsterdam and the other arriving by train from Brussels at approximately the same time. Let A and B be the events that the respective trains are on time. Suppose we

know from past experience that $P(A) = 0.93$, $P(B) = 0.89$, and $P(A \cap B) = 0.87$. Then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.93 + 0.89 - 0.87 = 0.95$$

is the probability that at least one train is on time.   ■

THEOREM
1.2-6

If A, B, and C are any three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$
$$- P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Proof. Write

$$A \cup B \cup C = A \cup (B \cup C)$$

and apply Theorem 1.2-5. The details are left as an exercise.   □

EXAMPLE 1.2-5   A survey was taken of a group's viewing habits of sporting events on TV during the last year. Let $A = \{\text{watched football}\}$, $B = \{\text{watched basketball}\}$, $C = \{\text{watched baseball}\}$. The results indicate that if a person is selected from the group surveyed, then $P(A) = 0.43$, $P(B) = 0.40$, $P(C) = 0.32$, $P(A \cap B) = 0.29$, $P(A \cap C) = 0.22$, $P(B \cap C) = 0.20$, and $P(A \cap B \cap C) = 0.15$. It then follows that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$$
$$- P(B \cap C) + P(A \cap B \cap C)$$
$$= 0.43 + 0.40 + 0.32 - 0.29 - 0.22 - 0.20 + 0.15$$
$$= 0.59$$

is the probability that this person watched at least one of these sports.   ■

Let a probability set function be defined on a sample space S. Let $S = \{e_1, e_2, \ldots, e_m\}$, where each e_i is a possible outcome of the experiment. The integer m is called the total number of ways in which the random experiment can terminate. If each of these outcomes has the same probability of occurring, we say that the m outcomes are **equally likely**. That is,

$$P(\{e_i\}) = \frac{1}{m}, \qquad i = 1, 2, \ldots, m.$$

If the number of outcomes in an event A is h, then the integer h is called the number of ways that are favorable to the event A. In this case, $P(A)$ is equal to the number of ways favorable to the event A divided by the total number of ways in which the experiment can terminate. That is, under this assumption of equally likely outcomes, we have

$$P(A) = \frac{h}{m} = \frac{N(A)}{N(S)},$$

where $h = N(A)$ is the number of ways A can occur and $m = N(S)$ is the number of ways S can occur. Exercise 1.2-19 considers this assignment of probability in a more theoretical manner.

It should be emphasized that in order to assign the probability h/m to the event A, we must assume that each of the outcomes $e_1, e_2, \ldots, e_m$ has the same probability $1/m$. This assumption is then an important part of our probability model; if it is not realistic in an application, then the probability of the event A cannot be computed in this way. Actually, we have used this result in the simple case given in Example 1.2-3 because it seemed realistic to assume that each of the possible outcomes in $S = \{HH, HT, TH, TT\}$ had the same chance of being observed.

EXAMPLE 1.2-6 Let a card be drawn at random from an ordinary deck of 52 playing cards. Then the sample space S is the set of $m = 52$ different cards, and it is reasonable to assume that each of these cards has the same probability of selection, $1/52$. Accordingly, if A is the set of outcomes that are kings, then $P(A) = 4/52 = 1/13$ because there are $h = 4$ kings in the deck. That is, $1/13$ is the probability of drawing a card that is a king, provided that each of the 52 cards has the same probability of being drawn. ∎

In Example 1.2-6, the computations are very easy because there is no difficulty in the determination of the appropriate values of h and m. However, instead of drawing only one card, suppose that 13 are taken at random and without replacement. Then we can think of each possible 13-card hand as being an outcome in a sample space, and it is reasonable to assume that each of these outcomes has the same probability. For example, to use the preceding method to assign the probability of a hand consisting of seven spades and six hearts, we must be able to count the number h of all such hands, as well as the number m of possible 13-card hands. In these more complicated situations, we need better methods of determining h and m. We discuss some of these counting techniques in Section 1.3.

EXERCISES

1.2-1. Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

1.2-2. An insurance company looks at its auto insurance customers and finds that (a) all insure at least one car, (b) 85% insure more than one car, (c) 23% insure a sports car, and (d) 17% insure more than one car, including a sports car. Find the probability that a customer selected at random insures exactly one car and it is not a sports car.

1.2-3. Draw one card at random from a standard deck of cards. The sample space S is the collection of the 52 cards. Assume that the probability set function assigns $1/52$ to each of the 52 outcomes. Let

$A = \{x : x \text{ is a jack, queen, or king}\}$,

$B = \{x : x \text{ is a } 9, 10, \text{ or jack and } x \text{ is red}\}$,

$C = \{x : x \text{ is a club}\}$,

$D = \{x : x \text{ is a diamond, a heart, or a spade}\}$.

Find (a) $P(A)$, (b) $P(A \cap B)$, (c) $P(A \cup B)$, (d) $P(C \cup D)$, and (e) $P(C \cap D)$.

1.2-4. A coin is tossed four times, and the sequence of heads and tails is observed.

(a) List each of the 16 sequences in the sample space S.

(b) Let events A, B, C, and D be given by $A = \{\text{at least 3 heads}\}$, $B = \{\text{at most 2 heads}\}$, $C = \{\text{heads on the third toss}\}$, and $D = \{1 \text{ head and 3 tails}\}$. If the probability set function assigns $1/16$ to each outcome in the sample space, find (i) $P(A)$, (ii) $P(A \cap B)$, (iii) $P(B)$,

(iv) $P(A \cap C)$, (v) $P(D)$, (vi) $P(A \cup C)$, and (vii) $P(B \cap D)$.

1.2-5. A field of beans is planted with three seeds per hill. For each hill of beans, let A_i be the event that i seeds germinate, $i = 0, 1, 2, 3$. Suppose that $P(A_0) = 1/64$, $P(A_1) = 9/64$, and $P(A_2) = 27/64$. Give the value of $P(A_3)$.

1.2-6. Consider the trial on which a 3 is first observed in successive rolls of a six-sided die. Let A be the event that 3 is observed on the first trial. Let B be the event that at least two trials are required to observe a 3. Assuming that each side has probability 1/6, find (a) $P(A)$, (b) $P(B)$, and (c) $P(A \cup B)$.

1.2-7. A fair eight-sided die is rolled once. Let $A = \{2, 4, 6, 8\}$, $B = \{3, 6\}$, $C = \{2, 5, 7\}$, and $D = \{1, 3, 5, 7\}$. Assume that each face has the same probability.

   (a) Give the values of (i) $P(A)$, (ii) $P(B)$, (iii) $P(C)$, and (iv) $P(D)$.

   (b) Give the values of (i) $P(A \cap B)$, (ii) $P(B \cap C)$, and (iii) $P(C \cap D)$.

   (c) Give the values of (i) $P(A \cup B)$, (ii) $P(B \cup C)$, and (iii) $P(C \cup D)$, using Theorem 1.2-5.

1.2-8. If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find (a) $P(A \cup B)$, (b) $P(A \cap B')$, and (c) $P(A' \cup B')$. *For (b) use event decomp. method.*

1.2-9. Given that $P(A \cup B) = 0.76$ and $P(A \cup B') = 0.87$, find $P(A)$.

1.2-10. During a visit to a primary care physician's office, the probability of having neither lab work nor referral to a specialist is 0.21. Of those coming to that office, the probability of having lab work is 0.41 and the probability of having a referral is 0.53. What is the probability of having both lab work and a referral?

1.2-11. Roll a fair six-sided die three times. Let $A_1 = \{1 \text{ or } 2 \text{ on the first roll}\}$, $A_2 = \{3 \text{ or } 4 \text{ on the second roll}\}$, and $A_3 = \{5 \text{ or } 6 \text{ on the third roll}\}$. It is given that $P(A_i) = 1/3$, $i = 1, 2, 3$; $P(A_i \cap A_j) = (1/3)^2, i \neq j$; and $P(A_1 \cap A_2 \cap A_3) = (1/3)^3$.

   (a) Use Theorem 1.2-6 to find $P(A_1 \cup A_2 \cup A_3)$.

   (b) Show that $P(A_1 \cup A_2 \cup A_3) = 1 - (1 - 1/3)^3$.

1.2-12. Prove Theorem 1.2-6.

1.2-13. For each positive integer n, let $P(\{n\}) = (1/2)^n$. Consider the events $A = \{n : 1 \leq n \leq 10\}$, $B = \{n : 1 \leq n \leq 20\}$, and $C = \{n : 11 \leq n \leq 20\}$. Find (a) $P(A)$, (b) $P(B)$, (c) $P(A \cup B)$, (d) $P(A \cap B)$, (e) $P(C)$, and (f) $P(B')$.

1.2-14. Let x equal a number that is selected randomly from the closed interval from zero to one, $[0, 1]$. Use your intuition to assign values to

   (a) $P(\{x : 0 \leq x \leq 1/3\})$.

   (b) $P(\{x : 1/3 \leq x \leq 1\})$.

   (c) $P(\{x : x = 1/3\})$.

   (d) $P(\{x : 1/2 < x < 5\})$.

1.2-15. A typical roulette wheel used in a casino has 38 slots that are numbered $1, 2, 3, \ldots, 36, 0, 00$, respectively. The 0 and 00 slots are colored green. Half of the remaining slots are red and half are black. Also, half of the integers between 1 and 36 inclusive are odd, half are even, and 0 and 00 are defined to be neither odd nor even. A ball is rolled around the wheel and ends up in one of the slots; we assume that each slot has equal probability of 1/38, and we are interested in the number of the slot into which the ball falls.

   (a) Define the sample space S.

   (b) Let $A = \{0, 00\}$. Give the value of $P(A)$.

   (c) Let $B = \{14, 15, 17, 18\}$. Give the value of $P(B)$.

   (d) Let $D = \{x : x \text{ is odd}\}$. Give the value of $P(D)$.

1.2-16. The five numbers 1, 2, 3, 4, and 5 are written respectively on five disks of the same size and placed in a hat. Two disks are drawn without replacement from the hat, and the numbers written on them are observed.

   (a) List the 10 possible outcomes of this experiment as unordered pairs of numbers.

   (b) If each of the 10 outcomes has probability 1/10, assign a value to the probability that the sum of the two numbers drawn is (i) 3; (ii) between 6 and 8 inclusive.

1.2-17. Divide a line segment into two parts by selecting a point at random. Use your intuition to assign a probability to the event that the longer segment is at least two times longer than the shorter segment.

1.2-18. Let the interval $[-r, r]$ be the base of a semicircle. If a point is selected at random from this interval, assign a probability to the event that the length of the perpendicular segment from the point to the semicircle is less than $r/2$.

1.2-19. Let $S = A_1 \cup A_2 \cup \cdots \cup A_m$, where events $A_1, A_2, \ldots, A_m$ are mutually exclusive and exhaustive.

   (a) If $P(A_1) = P(A_2) = \cdots = P(A_m)$, show that $P(A_i) = 1/m$, $i = 1, 2, \ldots, m$.

(b) If $A = A_1 \cup A_2 \cup \cdots \cup A_h$, where $h < m$, and (a) holds, prove that $P(A) = h/m$.

1.2-20. Let p_n, $n = 0, 1, 2, \ldots$, be the probability that an automobile policyholder will file for n claims in a five-year period. The actuary involved makes the assumption that $p_{n+1} = (1/4)p_n$. What is the probability that the holder will file two or more claims during this period?

1.3 METHODS OF ENUMERATION

In this section, we develop counting techniques that are useful in determining the number of outcomes associated with the events of certain random experiments. We begin with a consideration of the multiplication principle.

Multiplication Principle: Suppose that an experiment (or procedure) E_1 has n_1 outcomes and, for each of these possible outcomes, an experiment (procedure) E_2 has n_2 possible outcomes. Then the composite experiment (procedure) $E_1 E_2$ that consists of performing first E_1 and then E_2 has $n_1 n_2$ possible outcomes.

EXAMPLE 1.3-1 Let E_1 denote the selection of a rat from a cage containing one female (F) rat and one male (M) rat. Let E_2 denote the administering of either drug A (A), drug B (B), or a placebo (P) to the selected rat. Then the outcome for the composite experiment can be denoted by an ordered pair, such as (F, P). In fact, the set of all possible outcomes, namely, $(2)(3) = 6$ of them, can be denoted by the following rectangular array:

$$(F, A) \quad (F, B) \quad (F, P)$$
$$(M, A) \quad (M, B) \quad (M, P)$$

Another way of illustrating the multiplication principle is with a tree diagram like that in Figure 1.3-1. The diagram shows that there are $n_1 = 2$ possibilities (branches) for the sex of the rat and that, for each of these outcomes, there are $n_2 = 3$ possibilities (branches) for the drug.
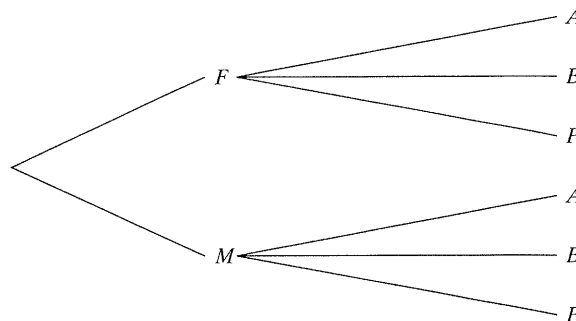

FIGURE 1.3-1: Tree diagram

Clearly, the multiplication principle can be extended to a sequence of more than two experiments or procedures. Suppose that the experiment E_i has n_i ($i = 1, 2, \ldots, m$) possible outcomes after previous experiments have been performed. Then the composite experiment $E_1 E_2 \cdots E_m$ that consists of performing E_1, then $E_2, \ldots,$ and finally E_m has $n_1 n_2 \cdots n_m$ possible outcomes.

EXAMPLE 1.3-2 A certain food service gives the following choices for dinner: E_1, soup or tomato juice; E_2, steak or shrimp; E_3, French fried potatoes, mashed potatoes, or a baked