Announcements

1) HW 1 due next Tuesday

Notation: $(\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N})$

$\mathbb{N}$ = natural numbers

$1, 2, 3, 4, \ldots$

$\mathbb{Z}$ = $\mathbb{N}$ along with its negatives and zero

$\ldots -2, -1, 0, 1, 2, \ldots$

$\mathbb{Q}$ = all quotients $\frac{a}{b}$ with a, b in $\mathbb{Z}$ and $b \neq 0$

$\mathbb{R}$ = all real numbers

Statements

(Section 1.1)

A statement (or proposition) will be a declarative sentence that is either true or false, but not both.

Example 1:

a) "Professor Wiggins weighs 300 pounds" is a statement.

b) "Does Professor Wiggins weigh 300 pounds?" is not declarative, so not a statement.

c) "Professor Wiggins should stop discussing his weight" is an opinion and therefore not a statement

Our main goal will be learning how to decide whether statements are (provably) true or (provably) false. Special emphasis will be placed on catching lies!

We will most often encounter **conditional statements**: a statement of the form

"If P, then Q"

where P and Q are sentences.

P is called the **hypothesis** and Q is called the **conclusion**

Example 2:

a) "If Rafael Nadal wins the US Open, then he will be the number-one ranked tennis player" is a conditional statement.

b) "Horses have infinitely many legs" is a statement, but not conditional.

You can see how the validity of P and Q affect a conditional statement using a truth table.

P	Q	If P, then Q
T	T	T
T	F	F
F	T	T
F	F	T

"T" = True

"F" = False

Example 3: Take

"If n is an even prime number, then $n=2$"

If both "n is an even prime" and "$n=2$" are true, then this statement is true.

If n can be an even prime but $n \neq 2$, this statement is false.

The interesting part is: what if n is not an even prime number? Then we can conclude anything we want using "If n is an even prime number" as our hypothesis.

This makes the conditional into what is called a vacuously true statement: it is true, but has no content.

Class Moral: When you are trying to prove a statement, never assume the conclusion!

Direct Proofs

(Section 1.2)

In time, we will discuss strategies for proving statements to be true or false. When you have none of these, you go for a direct proof - straight from the hypothesis using agreed-upon results!

Notation: $2\mathbb{Z}$ = even integers.

An integer m is even if there is an n in $\mathbb{Z}$, $m = 2n$.

$\mathbb{Z} - 2\mathbb{Z}$ will be the odd integers. An integer k is odd if there is an n in $\mathbb{Z}$, $k = 2n + 1$.

Proposition: The difference between squares of consecutive natural numbers is always an odd integer.

prelude What do we mean by "difference"? What do we mean by "squares"? What do we mean by "consecutive"? For how many natural numbers does this proposition claim to apply? How should we represent these numbers?

Proof: Let n be a natural number. The next consecutive number is $n+1$. Now calculate the difference between the squares:

$$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2$$

$$= 2n+1$$ which is odd. Not quite enough!

$$n^2 - (n+1)^2 = n^2 - (n^2+2n+1)$$

$$= n^2 - n^2 - 2n - 1$$

$$= -2n - 1$$

Not quite of the form we want, but easily fixed:

$$-2n-1 = 2(-n-1/2)$$

$$= 2(-n\underbrace{-1+1}_{=0}-1/2)$$

$$= 2(-n-1+1/2)$$

$$= 2(-n-1)+1 \quad \square$$

Write something to let people know your proof is finished!