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2.4 If A and B are two sets, draw Venn diagrams to verify the following:

a $A = (A \cap B) \cup (A \cap \overline{B})$.

b If $B \subset A$ then $A = B \cup (A \cap \overline{B})$.

2.5 Refer to Exercise 2.4. Use the identities $A = A \cap S$ and $S = B \cup \overline{B}$ and a distributive law to prove that

a $A = (A \cap B) \cup (A \cap \overline{B})$.

b If $B \subset A$ then $A = B \cup (A \cap \overline{B})$.

c Further, show that $(A \cap B)$ and $(A \cap \overline{B})$ are mutually exclusive and therefore that A is the union of two mutually exclusive sets, $(A \cap B)$ and $(A \cap \overline{B})$.

d Also show that B and $(A \cap \overline{B})$ are mutually exclusive and if $B \subset A$, A is the union of two mutually exclusive sets, B and $(A \cap \overline{B})$.

2.6 Suppose two dice are tossed and the numbers on the upper faces are observed. Let S denote the set of all possible pairs that can be observed. [These pairs can be listed, for example, by letting (2, 3) denote that a 2 was observed on the first die and a 3 on the second.]

a Define the following subsets of S:

A: The number on the second die is even.
B: The sum of the two numbers is even.
C: At least one number in the pair is odd.

b List the points in A, $\overline{C}$, $A \cap B$, $A \cap \overline{B}$, $\overline{A} \cup B$, and $\overline{A} \cap C$.

2.7 A group of five applicants for a pair of identical jobs consists of three men and two women. The employer is to select two of the five applicants for the jobs. Let S denote the set of all possible outcomes for the employer's selection. Let A denote the subset of outcomes corresponding to the selection of two men and B the subset corresponding to the selection of at least one woman. List the outcomes in A, $\overline{B}$, $A \cup B$, $A \cap B$, and $A \cap \overline{B}$. (Denote the different men and women by M_1, M_2, M_3 and W_1, W_2, respectively.)

2.8 From a survey of 60 students attending a university, it was found that 9 were living off campus, 36 were undergraduates, and 3 were undergraduates living off campus. Find the number of these students who were

a undergraduates, were living off campus, or both.

b undergraduates living on campus.

c graduate students living on campus.

2.4 A Probabilistic Model for an Experiment: The Discrete Case

DEFINITION 2.2

In Section 2.2 we referred to the die-tossing *experiment* when we observed the number appearing on the upper face. We will use the term *experiment* to include observations obtained from completely uncontrollable situations (such as observations on the daily price of a particular stock) as well as those made under controlled laboratory conditions. We have the following definition:

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succinctly. Hence, our next topic concerns some elementary results in combinatorial analysis and their application to the sample-point approach for the solution of probability problems.

Exercises

2.25 A single car is randomly selected from among all of those registered at a local tag agency. What do you think of the following claim? "All cars are either Volkswagens or they are not. Therefore, the probability is 1/2 that the car selected is a Volkswagen."

2.26 According to *Webster's New Collegiate Dictionary*, a divining rod is "a forked rod believed to indicate [divine] the presence of water or minerals by dipping downward when held over a vein." To test the claims of a divining rod expert, skeptics bury four cans in the ground, two empty and two filled with water. The expert is led to the four cans and told that two contain water. He uses the divining rod to test each of the four cans and decide which two contain water.

a List the sample space for this experiment.

b If the divining rod is completely useless for locating water, what is the probability that the expert will correctly identify (by guessing) both of the cans containing water?

2.27 In Exercise 2.12 we considered a situation where cars entering an intersection each could turn right, turn left, or go straight. An experiment consists of observing two vehicles moving through the intersection.

a How many sample points are there in the sample space? List them.

b Assuming that all sample points are equally likely, what is the probability that at least one car turns left?

c Again assuming equally likely sample points, what is the probability that at most one vehicle turns?

2.28 Four equally qualified people apply for two identical positions in a company. One and only one applicant is a member of a minority group. The positions are filled by choosing two of the applicants at random.

a List the possible outcomes for this experiment.

b Assign reasonable probabilities to the sample points.

c Find the probability that the applicant from the minority group is selected for a position.

2.29 Two additional jurors are needed to complete a jury for a criminal trial. There are six prospective jurors, two women and four men. Two jurors are randomly selected from the six available.

a Define the experiment and describe one sample point. Assume that you need describe only the two jurors chosen and not the order in which they were selected.

b List the sample space associated with this experiment.

c What is the probability that both of the jurors selected are women?

2.30 Three imported wines are to be ranked from lowest to highest by a purported wine expert. That is, one wine will be identified as best, another as second best, and the remaining wine as worst.

a Describe one sample point for this experiment.

b List the sample space.

c Assume that the "expert" really knows nothing about wine and randomly assigns ranks to the three wines. One of the wines is of much better quality than the others. What is the probability that the expert ranks the best wine no worse than second best?