$\boxed{\text{Supplement}}$ The Geometric Probability Distribution

Say: Sometimes one is interested in repeating a trial over and over again until a success occurs. In such a situation, one is interested in the probability that a success will occur on the n-th trial. For instance, if the probability of an engine malfunction during any one hour period is .10, let Y be the random variable which counts how many hours go by until the first malfunction. How many hours would you expect to wait until the first malfunction? The expected value of Y.

Def A geometric experiment consists of

- independent and identical trials repeated until success occurs
- each trial has only two possible outcomes: success or failure
- the prob of success on any trial is p
- the random var. Y is the trial number which is success

sample pt	Y	prob of sample pt
S	1	p
FS	2	qp
FFS	3	$qqp = q^2p$
FFFS	4	$qqqp = q^3p$
FFFFS	5	$qqqqp = q^4p$
⋮	⋮	⋮

$\Rightarrow$ sample space is countably infinite, and range Y = all positive integers.

The prob. mass function is

y	$p(y) = P(Y=y)$
1	p
2	qp
3	q^2p
4	q^3p
$\vdots$	$\vdots$

Def A <u>geometric probability distribution</u> is $p(y) = q^{y-1}p$ w/ dom p = positive integers and $0 < p \leq 1$, $q = 1-p$.

Rem $\sum_{y=1}^{\infty} p(y) = \sum_{y=1}^{\infty} q^{y-1}p = p \cdot \sum_{y=0}^{\infty} (1-p)^y = p \cdot \frac{1}{1-(1-p)} = 1$.

as we expect of a p.m.f.

If $p > 0$, we know a success will eventually occur w/ prob. 1.

Def Any random var. w/ p.m.f. $p(y) = q^{y-1}p$ and $0 < p \leq 1$ is called a <u>geometric random variable</u>.

Thm Y geometric $\Longrightarrow E[Y] = \frac{1}{p}$, $V(Y) = \frac{1-p}{p^2}$.

Ex Probs 3.73 and 3.74 in WMS. Success := an audited company has substantial errors.

$p = .9$ $\quad Y =$ no. of accounts audited until the first w/ subst. errors.

WMS #3.73

a) $p(3) = P(Y=3) = (.1)^2(.9) = .009$ FFS

b) $P(Y \geq 3) = \sum_{y=3}^{\infty} p(y) = 1 - \sum_{y=1}^{2} p(y)$

$= 1 - (.9) - (.1)(.9) = .01$

or note FF

$P(Y\geq 3) = P(Y>2)$
$= (.1)(.1)$
$= .01$

For comparison $P(Y \leq 2) = P((Y=1) \cup (Y=2)) = P(Y=1) + P(Y=2)$
$= p(1) + p(2)$
$= .9 + (.1)(.9) = .99$

So to find a co. w/ subst. errors, all the CPA needs to do is audit two accounts.

WMS #3.74

$E[Y] = \frac{1}{p} = \frac{1}{.9} = 1.\bar{1}$

$V(Y) = \frac{1-p}{p^2} = \frac{1-.9}{.9^2} = \frac{.1}{.81} = .123\ldots$

$SD(Y) = \sqrt{V(Y)} = \sqrt{.123\ldots} \approx .35$