Significant Figures in Calculations

Measurements always have some uncertainty. The degree of uncertainty can be expressed with the measured value, for example, 16.2 ±0.05 cm. This would indicate that the measurement is about 16.2 cm, but certainly not more than 16.25 cm or less than 16.15 cm. Measurements without the ± notation to indicate uncertainty are assumed to be uncertain in accordance with standard rules for significant figures. In this case the measurement 16.2 cm would indicate that the measurement was estimated to the closest one/tenth centimeter (0.1 cm) and that it is closer to 16.2 cm than it is to 16.1 cm or 16.3 cm. The last digit in the measurement has implied uncertainty.

1. Any measurement should be expressed to the number of significant figures consistent with the precision of the measurement. A measurement made to the nearest gram as 3. grams should not be expressed as 3.0 g or 3.00 g. Or a measurement read to 3.00 g should not be written as 3 g, or 3.0 g.

2. The **number of significant figures** is determined by counting the first nonzero digit and all following digits in a number with a decimal. *In the United States use a period (.)* ***not*** *a comma (,) for a decimal point.* Thus:

   a. Zeros before the first nonzero digit are **never** significant. 0.0034 has 2 sig figs.

   b. Internal zeros (zeros between nonzero digits) are **always** significant. 0.040067 has 5 sig figs.

   c. Final zeros **in a decimal number** are always significant. 4.500 has 4 sig figs.

3. Numbers without decimals may be:

   a. Exact: 1 mile = 5280 feet (by definition)

   b. Countable: 1 ream = 500 sheets

   c. Approximate: The distance to the sun is 93,000,000 miles

   Exact and countable numbers should be written without decimals.

   Usually measurements are best written so as to indicate the correct number of significant figures. It is always possible to write a number in exponential notation with an unambiguous number of significant figures. The distance to the sun is 9.3×10^7 mi. in scientific notation. Notice in exponential notation the decimal number (9.3) carries the correct number of significant figures. Significant figures do not depend upon the exponent (7 in 10^7).

   Scientific notation is exponential notation with one (only one) nonzero digit before the decimal. $93.\times 10^6$ mi. is exponential notation, but not scientific notation. It is usually best to express decimal numbers in scientific notation when standard notation is insufficient for expressing correct significant figures or when numbers are less than 0.01 or greater than 9999.

4. **In multiplication and division** the calculated answer should have no more significant figures than the measurement with the least significant figures. $0.044 \times 134.56 = 5.92064 \rightarrow 5.9$ (2 sig figs)

a. Exact numbers and countable numbers do not limit significant figures.

$$4.675 \text{ ft} = 4.675 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} = 56.10 \text{ in} \quad \text{(4 sig figs according to the original measurement)}$$

By definition there are 12 inches in one foot. This is not determined by measurement.

b. Conversion factors with decimals should carry at least as many significant figures as the number of significant figures in the measurements.

$$153.47 \text{ g} = 153.47 \text{ g} \times \frac{1 \text{ lb}}{454. \text{ g}} = 0.338 \text{ lb} \qquad 153.47 \text{ g} = 153.47 \text{ g} \times \frac{1 \text{ lb}}{453.592 \text{ g}} = 0.33834 \text{ lb}$$

The conversion 1 1b = 454. g is only good to 3 sig figs. Calculations using a conversion factor with 3 sig figs will not be better than 3 sig figs. It is a mistake to use a conversion factor with less significant figures than the measurements being converted. It is acceptable and usually desirable to use more sig figs in the conversion factor than the minimum.

5. **In addition and subtraction** the calculated answer should carry as many **decimal places** (not significant figures) as the measurement with the fewest decimal places.

$$\begin{array}{r} 453.1 \\ +\ 1.23 \\ \hline 454.33 \end{array} \rightarrow 454.3 \qquad \begin{array}{r} 453.1122 \\ +\ 1.23 \\ \hline 454.3422 \end{array} \rightarrow 454.34 \qquad \begin{array}{r} 453. \\ +\ 0.034567 \\ \hline 453.034567 \end{array} \rightarrow 453.$$

6. **Round the answer** to the nearest number having the correct number of significant figures.

a. If the leftmost digit being removed is less than 5, drop this digit and all following digits.

b. If the leftmost digit being removed is 5 or greater, drop this digit and all following digits AND increase that rightmost digit being retained by one unit.

Round to	7 sig figs	6 sig figs	5 sig figs	4 sig figs	3 sig figs	2 sig figs	1 sig fig
457.26451	457.2645	457.265	457.26	457.3	457.	4.6×10^2	$5. \times 10^2$

c. Round from the original number, not a number derived by rounding. E.g. 457.26451 → 457.26

d. Notice that rounding cannot change the magnitude of the number. A number between 400 and 500 before rounding must still be between 400 and 500 after rounding. Convert the number to scientific notation before rounding whenever convenient or necessary. $457.26451 = 4.5726451 \times 10^2$

e. Calculate an answer to a problem using all given significant figures, then round the **final** answer to the correct significant figures. Consider each step along the way to determine correct significant figures, but <u>do not round at each step along the way</u>.