MATH 425/525, REVIEW OF DISCRETE RANDOM VARIABLES, BY PROF. T. FIORE

Name: _____

Date: _____

The Notion of Random Variable, a Discrete Example, its PMF and CDF

1. What is the mathematical definition of *random variable*? Your answer must be a complete sentence with proper mathematical notation. As is standard practice in mathematics, the word you are defining must be underlined. Notice you are defining the general notion of random variable, not specifically discrete or continuous.

2. Now suppose that Y is a *discrete* random variable. What is the definition of the *probability mass function f* of Y? (No sentence needed) Use the usual math notation involving a colon and arrow and domain and codomain, and then give the formula for $f(y)$.

3. Consider the roll of two 3-sided dice, one is red and one is blue. After the roll, the number on the red die is recorded, and then the number on the blue die.

   (a) What is the *sample space*, or *outcome space* for this experiment?

   (b) Now consider the random variable Y defined by the maximum of the two numbers showing on the red and blue dice. What are the domain and codomain of Y? What is the formula that defines Y?

(c) Now consider the probability mass function f for the random variable Y above. What are the domain and codomain of f? Use the standard math arrow notation. Make a table which completely describes the pmf f. Show all work for your creation of the table. Begin with the definition of f at each valid input and compute it.

(d) Plot the probability mass function f in **two** ways.

(e) Let F be the cumulative distribution function of the random variable Y. What are the domain and codomain of F? Use standard math arrow notation. What is the general formula for a cumulative distribution function?

(f) Use your work above to determine the following.

$F(0) =$

$F(1) =$

$F(2) = \qquad\qquad F(3) =$

2

(g) Plot the cdf F. Pay attention to where the open and closed dots are.

Binomial Random Variables

4. Now consider a different random variable Y. Suppose Y has distribution $b(3, .75)$. In other words, suppose Y is a binomial random variable with $n = 3$ trials and success probability $p = .75$.

   (a) Write down the sample space.

   (b) Compute the following.

   $Y(SFF) =$ $\qquad$ $Y(SFS) =$ $\qquad$ $Y(FFF) =$ $\qquad$ $Y(SSS) =$

   (c) Write down a formula for the pmf f of Y. Your formula should involve no unknown parameters.

   (d) Let F be the cumulative distribution function of Y. Find $F(1.67)$ by hand.

3

(e) Which R command would you use to find $F(1.67)$?

(f) For a random variable with distribution $b(n, p)$, the mean (or expected value) is np while the variance is $np(1 - p)$. What are the mean and variance of the random variable Y above?

$\mu =$ $\sigma^2 =$

5. Give an example of a binomial experiment and its associated binomial random variable.

6. True or False: the sample space of every binomial random variable is finite.

7. True or False: the sample space of every discrete random variable is either finite or countably infinite.