Math 325, Some Final Exam Review

This is just a selection of some review problems, and is in no way representative of the final exam. For instance, these few problems do not include the later material at all.

(1) You ask a neighbor to water a sickly plant while you are away on vacation. Without water the plant will die with probability .7 . With water it will die with probability .2 . You are 90% certain the neighbor will remember to water the plant.

(a) Introduce notation for the relevant events and clearly say what letter stands for which event.

(b) Find the probability that the plant will die while you are away on vacation.

(2) What is the precise mathematical definition of a *random variable*?

(3) Suppose Y is a discrete random variable. What is the definition of the value of the *probability mass function for* Y below?

$p(y) =$

(4) Suppose X is a discrete random variable and its moment generating function is as below.

$$m(t) = \frac{1}{4}e^t + \frac{3}{4}e^{2t}$$

(a) Make a table for the probability mass function of X.

(b) Use the moment generating function to find the mean of X. If you do not know how to use the moment generating function to find the mean, then you may use your answer to 1(a) to find the mean for partial credit.

(5) Write True or False: Any exponential random variable is memoryless. (No work needed).

(6) Suppose X and Y are discrete random variables on the same sample space, that is, they are *jointly distributed*. Define the *joint probability mass function of* X *and* Y.

$p(x, y) =$

More on next page.

(7) Suppose X and Y are jointly distributed random variables. **Define** the *covariance of* X *and* Y.

$\mathrm{Cov}(X, Y) =$

To calculate the covariance of two jointly distributed random variables, we can use a simpler formula than the definition. What is that simpler formula?

$\mathrm{Cov}(X, Y) =$

If you forgot the definition or the simpler formula for covariance, say instead what the covariance of two jointly distributed random variables measures.

(8) Recall that the *Gamma distribution with shape parameter* α *and scale parameter* β is

$$f(y) = \frac{y^{\alpha-1}\, e^{-y/\beta}}{\beta^{\alpha}\,\Gamma(\alpha)}$$

for $y \geq 0$, and is zero for $y < 0$. Suppose that a random variable Y has a Gamma distribution with $f(y) = cy^2 e^{-2y}$ for some $c > 0$ when $y \geq 0$. Find α, β, and c.

$\alpha =$

$\beta =$

$c =$

Hint: to find c, do **not** try to integrate and set the integral equal to 1. You won't be able to find the integral. Instead, use the Gamma distribution.

(9) Suppose Y is a random variable (possibly discrete, possibly continuous). What is the definition of the *cumulative distribution function of* Y? Write a formula for $F(y)$ in terms of other quantities.

$F(y) =$

(10) Below, draw a graph of the *standard normal density*, label with a number the relevant maximum on the vertical axis, and label with numbers two relevant points on the horizontal axis to indicate the spread. Remember to label the axes.

(11) Suppose the joint mass function $p(x, y)$ of two discrete random variables X and Y is as follows.

$X \setminus Y$	4	5
1	.2	.3
2	.4	.1

(a) Compute the marginal mass function $p_Y(y)$.

$p_Y(4) =$

$p_Y(5) =$

(b) List the following set. Be sure to use proper set notation.

$(XY < 9) =$

(c) Compute: $P(XY < 9) =$

(12) Suppose two new random variables X and Y are measured on a population, and that all the pairs (x, y) lie on a line of slope $-1/3$. What is the correlation coefficient of the two random variables?

(13) In class we discussed two goals of statistics. Write a complete sentence which states one of these goals of statistics.

(14) The pharmaceutical company Procter & Gamble would like to sell a new blood pressure medication, but they need to first prove its effectiveness to the FDA. So they do a clinical trial. In this situation, what is the "population" and what is the "sample"?

population =

sample =

(15) Suppose the following five measurements are taken in some experiment: 10, 12, 18, 21, 30.

(a) Draw a **frequency histogram** such that the first bin begins at 9 and the bin width is 10.

Remember to **label the axes in words**.

(b) Considering the same set of measurements, how high is the bar above the bin $[9, 19]$ in a **relative** frequency histogram?