

Approximating the Heat Capacity of a Solution

To a first approximation the heat capacity of a mixture is the sum of the heat capacities of its parts. A solution is a mixture, so it makes sense to add the heat capacities of the solute and solvent to get the heat capacity of a solution. Dividing the total heat capacity by the total mass will give the specific heat of the solution.

See for example: <http://www.cheresources.com/estcp.shtml>

Consider a 2.00 % MgCl_2 (0.169 molar) solution. 100.0 g of solution would contain 98.0 g of water and 2.00 g of magnesium chloride. The heat capacity of the solution (100 g) would be:

$$98.00 \text{ g} \times 4.184 \frac{\text{J}}{\text{g} \cdot \text{C}^\circ} + 2.00 \times 0.7496 \frac{\text{J}}{\text{g} \cdot \text{C}^\circ} = 411.5 \text{ J/C}^\circ \quad \text{so} \quad \text{specific heat} = s = 4.115 \frac{\text{J}}{\text{g} \cdot \text{C}^\circ}$$

The heat necessary to increase the temperature of 100.0 g MgCl_2 solution (2%) by 20.0 C° is

$$q = m \times s \times \Delta T = 100.0 \text{ g} \times \frac{4.115 \text{ J}}{\text{g} \cdot \text{C}^\circ} \times 20 \text{ C}^\circ = 8231. \text{ J}$$

To approximate this solution as having a specific heat equal to that of water brings up the question of what to use for the mass of the solution. Using the mass of the whole 100.0 g of solution (mass of water + mass of solute) gives:

$$q = m \times s \times \Delta T = 100.0 \text{ g} \times \frac{4.184 \text{ J}}{\text{g} \cdot \text{C}^\circ} \times 20 \text{ C}^\circ = 8368. \text{ J}$$

$$\text{which is } \frac{8368. \text{ J} - 8231. \text{ J}}{8231. \text{ J}} \times 100\% = 1.66 \% \text{ too high}$$

However, by estimating that the heat change of the solution depends only upon the 98.00 g of water gives:

$$q = m \times s \times \Delta T = 98.00 \text{ g} \times \frac{4.184 \text{ J}}{\text{g} \cdot \text{C}^\circ} \times 20 \text{ C}^\circ = 8201. \text{ J}$$

$$\text{which is } \frac{8201. \text{ J} - 8231. \text{ J}}{8231. \text{ J}} \times 100\% = -0.36 \% \text{ or } 0.36\% \text{ too low}$$

This is a much better approximation.

When approximating the specific heat of a solution as the specific heat of water, one can expect to get the best results by using only the mass of the water in the solution (without including the mass of the salt) in the $q = ms\Delta T$ equation. An exception would occur when the specific heat of the solute is more than half of the specific heat of water.

The published density of a 2.00% MgCl_2 solution is 1.0144 g/mL. So 100.0 g MgCl_2 solution (2%) has volume 98.58 mL. Using the volume of the solution times 1g/mL as the approximate mass of water in the solution would lead to heat calculation:

$$q = m \times s \times \Delta T = 98.58 \text{ g} \times \frac{4.184 \text{ J}}{\text{g} \cdot \text{C}^\circ} \times 20 \text{ C}^\circ = 8249. \text{ J}$$

$$\text{This error is } \frac{8249. \text{ J} - 8231. \text{ J}}{8231. \text{ J}} \times 100\% = 0.21 \text{ \% error high}$$

This is a better approximation than using the mass of the water only. Including the volume change arising from adding the solute (and counting it as water) approximately compensates for neglecting the heat capacity of the added solute.

So a reasonably good approximation for the heat capacity of solution is to multiply the volume of the solution (in mL) times 1g/mL times 4.184 J/(g·C°)