- 2.4 If A and B are two sets, draw Venn diagrams to verify the following:
 - $\mathbf{a} \quad A = (A \cap B) \cup (A \cap \overline{B}).$
 - **b** If $B \subset A$ then $A = B \cup (A \cap \overline{B})$.
- **2.5** Refer to Exercise 2.4. Use the identities $A = A \cap S$ and $S = B \cup \overline{B}$ and a distributive law to prove that
 - $\mathbf{a} \quad A = (A \cap B) \cup (A \cap \overline{B}).$
 - **b** If $B \subset A$ then $A = B \cup (A \cap \overline{B})$.
 - **c** Further, show that $(A \cap B)$ and $(A \cap \overline{B})$ are mutually exclusive and therefore that A is the union of two mutually exclusive sets, $(A \cap B)$ and $(A \cap \overline{B})$.
 - **d** Also show that B and $(A \cap \overline{B})$ are mutually exclusive and if $B \subset A$, A is the union of two mutually exclusive sets, B and $(A \cap \overline{B})$.
- 2.6 Suppose two dice are tossed and the numbers on the upper faces are observed. Let S denote the set of all possible pairs that can be observed. [These pairs can be listed, for example, by letting (2, 3) denote that a 2 was observed on the first die and a 3 on the second.]
 - a Define the following subsets of S:
 - A: The number on the second die is even.
 - B: The sum of the two numbers is even.
 - C: At least one number in the pair is odd.
 - **b** List the points in $A, \overline{C}, A \cap B, A \cap \overline{B}, \overline{A} \cup B$, and $\overline{A} \cap C$.
- 2.7 A group of five applicants for a pair of identical jobs consists of three men and two women. The employer is to select two of the five applicants for the jobs. Let S denote the set of all possible outcomes for the employer's selection. Let A denote the subset of outcomes corresponding to the selection of two men and B the subset corresponding to the selection of at least one woman. List the outcomes in A, \overline{B} , $A \cup B$, $A \cap B$, and $A \cap \overline{B}$. (Denote the different men and women by M_1 , M_2 , M_3 and W_1 , W_2 , respectively.)
- 2.8 From a survey of 60 students attending a university, it was found that 9 were living off campus, 36 were undergraduates, and 3 were undergraduates living off campus. Find the number of these students who were
 - a undergraduates, were living off campus, or both,
 - **b** undergraduates living on campus.
 - c graduate students living on campus.

2.4 A Probabilistic Model for an Experiment: The Discrete Case

In Section 2.2 we referred to the die-tossing *experiment* when we observed the number appearing on the upper face. We will use the term *experiment* to include observations obtained from completely uncontrollable situations (such as observations on the daily price of a particular stock) as well as those made under controlled laboratory conditions. We have the following definition:

DEFINITION 2.2