Wackerly, Mendenhall, Scheaffer
Mathematical Statistics
with Applications

Some problems on
continuous random variables.

a Show that $F(y)$ has the properties of a distribution function.
b Find the .30-quantile, $\phi_{.30}$, of Y.
c Find $f(y)$.
d Find the probability that the transistor operates for at least 200 hours.
e Find $P(Y > 100 | Y \leq 200)$.

4.13 A supplier of kerosene has a 150-gallon tank that is filled at the beginning of each week. His weekly demand shows a relative frequency behavior that increases steadily up to 100 gallons and then levels off between 100 and 150 gallons. If Y denotes weekly demand in hundreds of gallons, the relative frequency of demand can be modeled by

$$f(y) = \begin{cases} y, & 0 \leq y \leq 1, \\ 1, & 1 < y \leq 1.5, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find $F(y)$.
b Find $P(0 \leq Y \leq .5)$.
c Find $P(.5 \leq Y \leq 1.2)$.

4.14 A gas station operates two pumps, each of which can pump up to 10,000 gallons of gas in a month. The total amount of gas pumped at the station in a month is a random variable Y (measured in 10,000 gallons) with a probability density function given by

$$f(y) = \begin{cases} y, & 0 < y < 1, \\ 2 - y, & 1 \leq y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

a Graph $f(y)$.
b Find $F(y)$ and graph it.
c Find the probability that the station will pump between 8000 and 12,000 gallons in a particular month.
d Given that the station pumped more than 10,000 gallons in a particular month, find the probability that the station pumped more than 15,000 gallons during the month.

4.15 As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \begin{cases} \dfrac{b}{y^2}, & y \geq b, \\ 0, & \text{elsewhere,} \end{cases}$$

where b is the minimum possible time needed to traverse the maze.

a Show that $f(y)$ has the properties of a density function.
b Find $F(y)$.
c Find $P(Y > b + c)$ for a positive constant c.
d If c and d are both positive constants such that $d > c$, find $P(Y > b + d | Y > b + c)$.

4.16 Let Y possess a density function

$$f(y) = \begin{cases} c(2 - y), & 0 \leq y \leq 2, \\ 0, & \text{elsewhere.} \end{cases}$$

THEOREM 4.6 If $\theta_1 < \theta_2$ and Y is a random variable uniformly distributed on the interval (θ_1, θ_2), then

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}.$$

Proof By Definition 4.5,

$$\begin{aligned} E(Y) &= \int_{-\infty}^{\infty} y f(y)\, dy \\ &= \int_{\theta_1}^{\theta_2} y \left(\frac{1}{\theta_2 - \theta_1}\right) dy \\ &= \left(\frac{1}{\theta_2 - \theta_1}\right) \frac{y^2}{2}\bigg]_{\theta_1}^{\theta_2} = \frac{\theta_2^2 - \theta_1^2}{2(\theta_2 - \theta_1)} \\ &= \frac{\theta_2 + \theta_1}{2}. \end{aligned}$$

Note that the mean of a uniform random variable is simply the value midway between the two parameter values, θ_1 and θ_2. The derivation of the variance is left as an exercise.

Problems on Uniform Random Variables

Exercises

4.38 Suppose that Y has a uniform distribution over the interval (0, 1).

- **a** Find $F(y)$.
- **b** Show that $P(a \le Y \le a + b)$, for $a \ge 0, b \ge 0,$ and $a + b \le 1$ depends only upon the value of b.

4.39 If a parachutist lands at a random point on a line between markers A and B, find the probability that she is closer to A than to B. Find the probability that her distance to A is more than three times her distance to B.

4.40 Suppose that three parachutists operate independently as described in Exercise 4.39. What is the probability that exactly one of the three lands past the midpoint between A and B?

4.41 A random variable Y has a uniform distribution over the interval (θ_1, θ_2). Derive the variance of Y.

4.42 The *median* of the distribution of a continuous random variable Y is the value $\phi_{.5}$ such that $P(Y \le \phi_{.5}) = 0.5$. What is the median of the uniform distribution on the interval (θ_1, θ_2)?

4.43 A circle of radius r has area $A = \pi r^2$. If a random circle has a radius that is uniformly distributed on the interval (0, 1), what are the mean and variance of the area of the circle?

4.44 The change in depth of a river from one day to the next, measured (in feet) at a specific location, is a random variable Y with the following density function:

$$f(y) = \begin{cases} k, & -2 \le y \le 2 \\ 0 & \text{elsewhere.} \end{cases}$$

a Determine the value of k.

b Obtain the distribution function for Y.

4.45 Upon studying low bids for shipping contracts, a microcomputer manufacturing company finds that intrastate contracts have low bids that are uniformly distributed between 20 and 25, in units of thousands of dollars. Find the probability that the low bid on the next intrastate shipping contract

a is below \$22,000.

b is in excess of \$24,000.

4.46 Refer to Exercise 4.45. Find the expected value of low bids on contracts of the type described there.

4.47 The failure of a circuit board interrupts work that utilizes a computing system until a new board is delivered. The delivery time, Y, is uniformly distributed on the interval one to five days. The cost of a board failure and interruption includes the fixed cost c_0 of a new board and a cost that increases proportionally to Y^2. If C is the cost incurred, $C = c_0 + c_1 Y^2$.

a Find the probability that the delivery time exceeds two days.

b In terms of c_0 and c_1, find the expected cost associated with a single failed circuit board.

4.48 If a point is *randomly* located in an interval (a, b) and if Y denotes the location of the point, then Y is assumed to have a uniform distribution over (a, b). A plant efficiency expert randomly selects a location along a 500-foot assembly line from which to observe the work habits of the workers on the line. What is the probability that the point she selects is

a within 25 feet of the end of the line?

b within 25 feet of the beginning of the line?

c closer to the beginning of the line than to the end of the line?

4.49 A telephone call arrived at a switchboard at random within a one-minute interval. The switch board was fully busy for 15 seconds into this one-minute period. What is the probability that the call arrived when the switchboard was not fully busy?

4.50 Beginning at 12:00 midnight, a computer center is up for one hour and then down for two hours on a regular cycle. A person who is unaware of this schedule dials the center at a random time between 12:00 midnight and 5:00 A.M. What is the probability that the center is up when the person's call comes in?

4.51 The cycle time for trucks hauling concrete to a highway construction site is uniformly distributed over the interval 50 to 70 minutes. What is the probability that the cycle time exceeds 65 minutes if it is known that the cycle time exceeds 55 minutes?

4.52 Refer to Exercise 4.51. Find the mean and variance of the cycle times for the trucks.

4.53 The number of defective circuit boards coming off a soldering machine follows a Poisson distribution. During a specific eight-hour day, one defective circuit board was found.

a Find the probability that it was produced during the first hour of operation during that day.

b Find the probability that it was produced during the last hour of operation during that day.

c Given that no defective circuit boards were produced during the first four hours of operation, find the probability that the defective board was manufactured during the fifth hour.

4.54 In using the triangulation method to determine the range of an acoustic source, the test equipment must accurately measure the time at which the spherical wave front arrives at a receiving

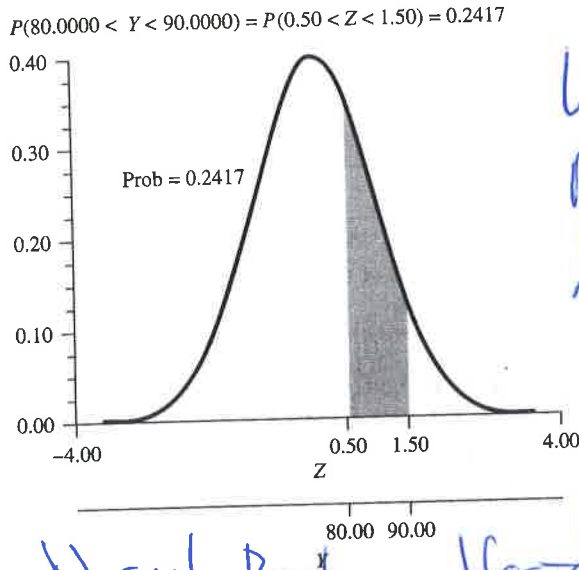

FIGURE 4.14 Required area for Example 4.9, using both the original and transformed (z) scales

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Problems on Normal Random Variables

d $P(-.2 \le Z \le .2)$

e $P(-1.56 \le Z \le -.2)$

f **Applet Exercise** Use the applet *Normal Probabilities* to obtain $P(0 \le Z \le 1.2)$. are the values given on the two horizontal axes identical?

4.59 If Z is a standard normal random variable, find the value z_0 such that

a $P(Z > z_0) = .5$.

b $P(Z < z_0) = .8643$.

c $P(-z_0 < Z < z_0) = .90$.

d $P(-z_0 < Z < z_0) = .99$.

4.60 A normally distributed random variable has density function

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}, \qquad -\infty < y < \infty.$$

Using the fundamental properties associated with any density function, argue that the par σ must be such that $\sigma > 0$.

4.61 What is the median of a normally distributed random variable with mean μ and st deviation σ?

4.62 If Z is a standard normal random variable, what is

a $P(Z^2 < 1)$?

b $P(Z^2 < 3.84146)$?

4.63 A company that manufactures and bottles apple juice uses a machine that automatica 16-ounce bottles. There is some variation, however, in the amounts of liquid dispensed bottles that are filled. The amount dispensed has been observed to be approximately n distributed with mean 16 ounces and standard deviation 1 ounce.

a Use Table 4, Appendix 3, to determine the proportion of bottles that will have more than 17 ounces dispensed into them.

b **Applet Exercise** Use the applet *Normal Probabilities* to obtain the answer to part (a).

4.64 The weekly amount of money spent on maintenance and repairs by a company was observed, over a long period of time, to be approximately normally distributed with mean \$400 and standard deviation \$20. If \$450 is budgeted for next week, what is the probability that the actual costs will exceed the budgeted amount?

a Answer the question, using Table 4, Appendix 3.

b **Applet Exercise** Use the applet *Normal Probabilities* to obtain the answer.

c Why are the labeled values different on the two horizontal axes?

4.65 In Exercise 4.64, how much should be budgeted for weekly repairs and maintenance to provide that the probability the budgeted amount will be exceeded in a given week is only .1?

4.66 A machining operation produces bearings with diameters that are normally distributed with mean 3.0005 inches and standard deviation .0010 inch. Specifications require the bearing diameters to lie in the interval $3.000 \pm .0020$ inches. Those outside the interval are considered scrap and must be remachined. With the existing machine setting, what fraction of total production will be scrap?

a Answer the question, using Table 4, Appendix 3.

b **Applet Exercise** Obtain the answer, using the applet *Normal Probabilities*.

4.67 In Exercise 4.66, what should the mean diameter be in order that the fraction of bearings scrapped be minimized?

4.68 The grade point averages (GPAs) of a large population of college students are approximately normally distributed with mean 2.4 and standard deviation .8. What fraction of the students will possess a GPA in excess of 3.0?

a Answer the question, using Table 4, Appendix 3.

b **Applet Exercise** Obtain the answer, using the applet *Normal Tail Areas and Quantiles*.

4.69 Refer to Exercise 4.68. If students possessing a GPA less than 1.9 are dropped from college, what percentage of the students will be dropped?

4.70 Refer to Exercise 4.68. Suppose that three students are randomly selected from the student body. What is the probability that all three will possess a GPA in excess of 3.0?

4.71 Wires manufactured for use in a computer system are specified to have resistances between .12 and .14 ohms. The actual measured resistances of the wires produced by company A have a normal probability distribution with mean .13 ohm and standard deviation .005 ohm.

a What is the probability that a randomly selected wire from company A's production will meet the specifications?

b If four of these wires are used in each computer system and all are selected from company A, what is the probability that all four in a randomly selected system will meet the specifications?

4.72 One method of arriving at economic forecasts is to use a consensus approach. A forecast is obtained from each of a large number of analysts; the average of these individual forecasts is the consensus forecast. Suppose that the individual 1996 January prime interest–rate forecasts of all economic analysts are approximately normally distributed with mean 7% and standard